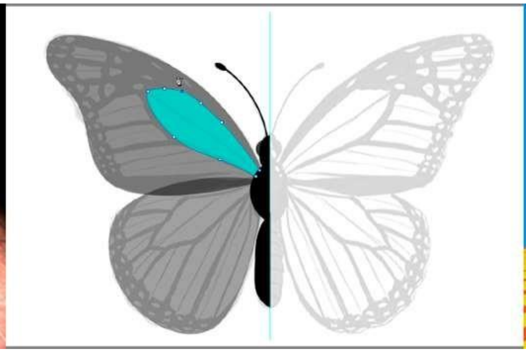
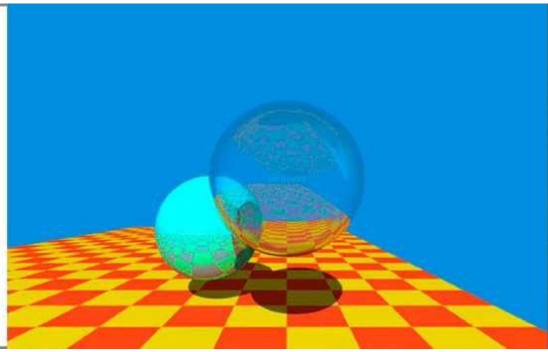
Computer Graphics

Transformation Cont.


Last Lecture

- Transformation
  - Why study transformation
  - 2D transformations: rotation, scale, shear
  - Homogeneous coordinates
  - Composite transform
  - 3D transformations

下面说法正确的是?

A
- vector+ vector = vector

B
- point – point = vector

C
- point + vector = vector

D
- Point + point = point

提交

Outline

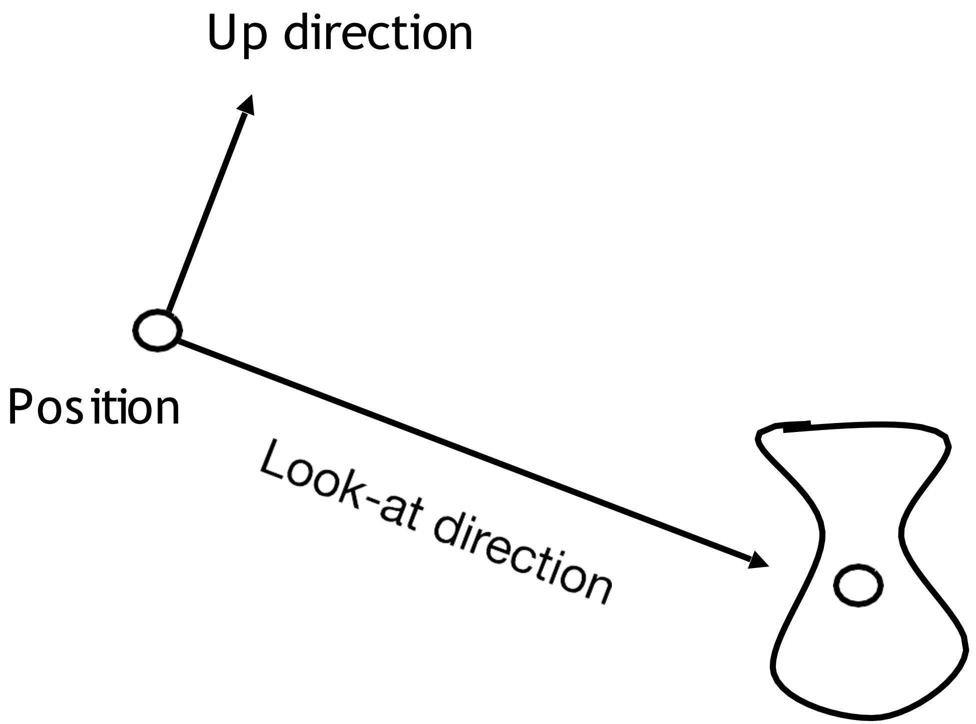
- Viewing transformation
  - View (视图)/ Camera transformation
  - Projection (投影) transformation
    - Orthographic (正交) projection
    - Perspective (透视) projection

View / Camera Transformation

- What is view transformation?
- Think about how to take a photo
  - Find a good place and arrange people (model transformation)
  - Find a good "angle" to put the camera (view transformation)
  - Cheese! (projection transformation)

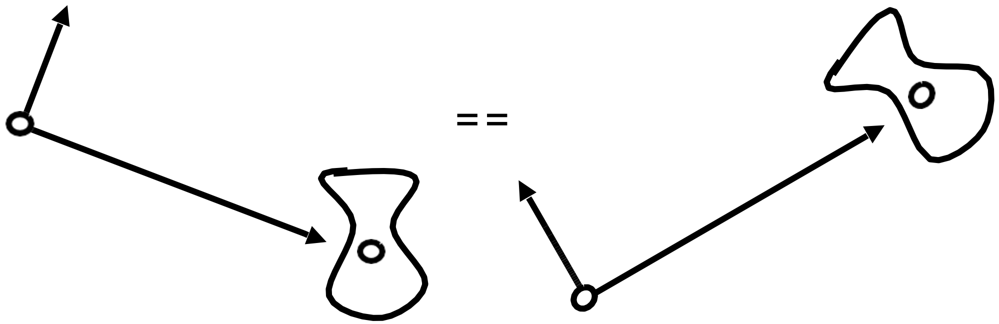
View / Camera Transformation

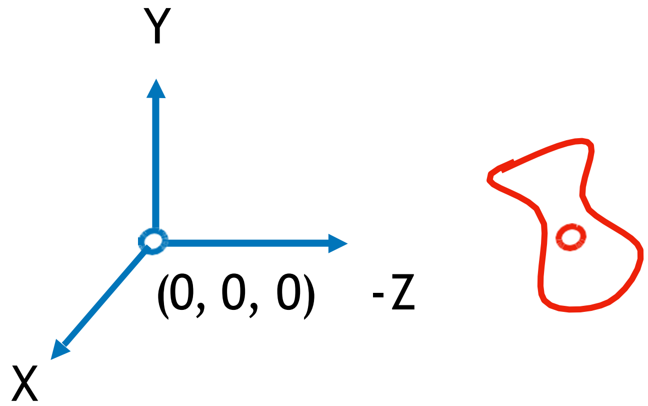
- How to perform view transformation?
- Define the camera first
  - Position $\vec{e}$
  - Look-at / gaze direction $\hat{g}$
  - Up direction $\hat{t}$
    (assuming perp. to look-at)



View / Camera Transformation

- Key observation
  - If the camera and all objects move together, the "photo" will be the same



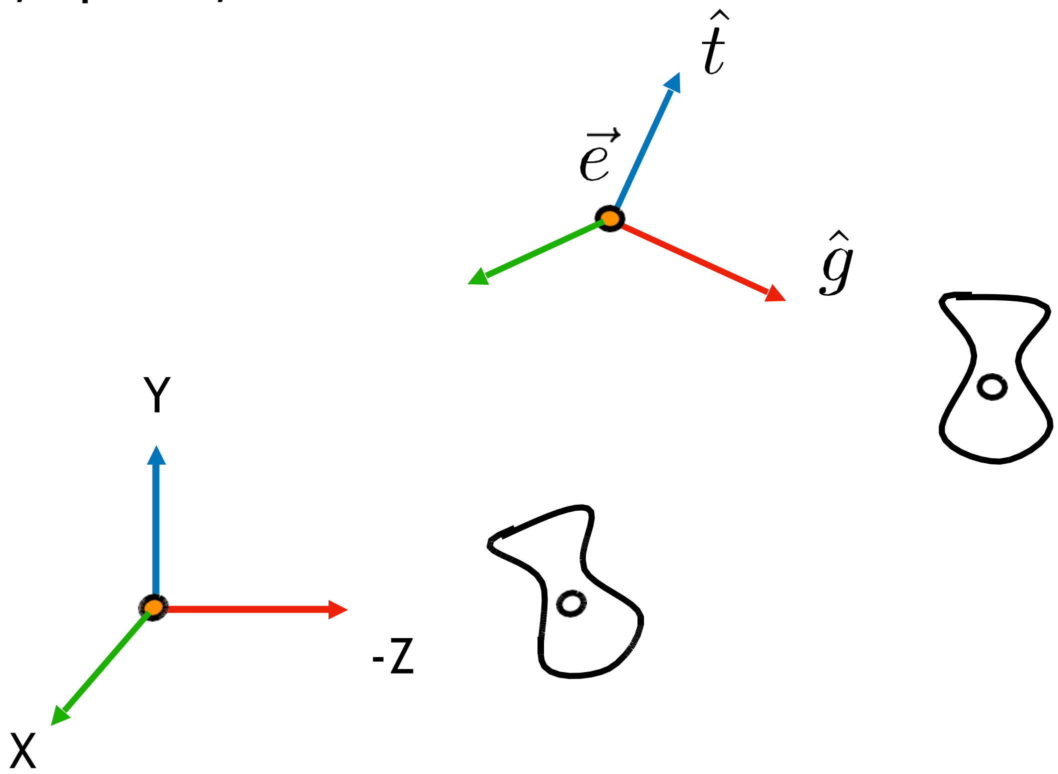


- How about that we always transform the camera to
  - The origin, up at Y, look at -Z
  - And transform the objects along with the camera

View / Camera Transformation

- Transform the camera by M_{view}
  - So it's located at the origin, up at Y, look at -Z

- M_{view} in math?
  - Translates e to origin
  - Rotates g to -Z
  - Rotates t to Y
  - Rotates (g x t) To X
  - Difficult to write!



View / Camera Transformation

- M_{view} in math?
  - Let's write $M_{view} = R_{view}T_{view}$
  - Translate e to origin

$$T_{view} = \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - Rotate g to -Z, t to Y, (g x t) To X
  - Consider its inverse rotation: X to (g x t), Y to t, Z to -g

$$R_{view}^{-1} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & x_t & x_{-g} & 0 \\ y_{\hat{g}\times\hat{t}} & y_t & y_{-g} & 0 \\ z_{\hat{g}\times\hat{t}} & z_t & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

WHY ?

$$R_{view} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & y_{\hat{g}\times\hat{t}} & z_{\hat{g}\times\hat{t}} & 0 \\ x_t & y_t & z_t & 0 \\ x_{-g} & y_{-g} & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

View / Camera Transformation

- Summary
  - Transform objects together with the camera
  - Until camera's at the origin, up at Y, lookat -Z
- Also known as ModelView Transformation
- But why do we need this?
  - For projection transformation!

Outline

- Viewing transformation
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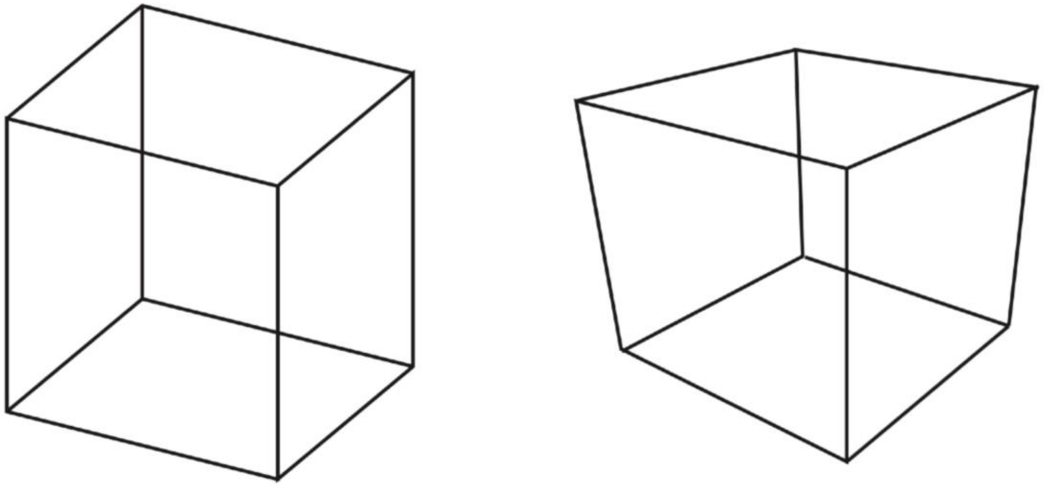
    - Orthographic (正交) projection
    - Perspective (透视) projection

Projection Transformation

- Projection in Computer Graphics
  - 3D to 2D
  - Orthographic projection
  - Perspective projection

Orthographic projection

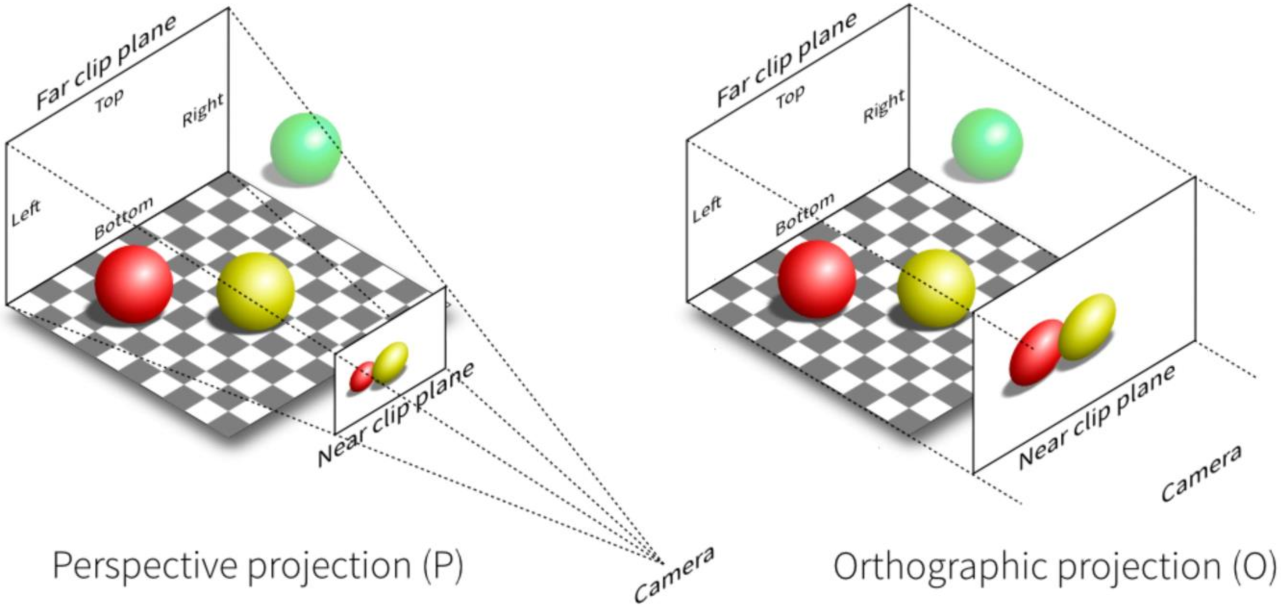


Perspective projection

Fig. 7.1 from *Fundamentals of Computer Graphics, 4th Edition*

Projection Transformation

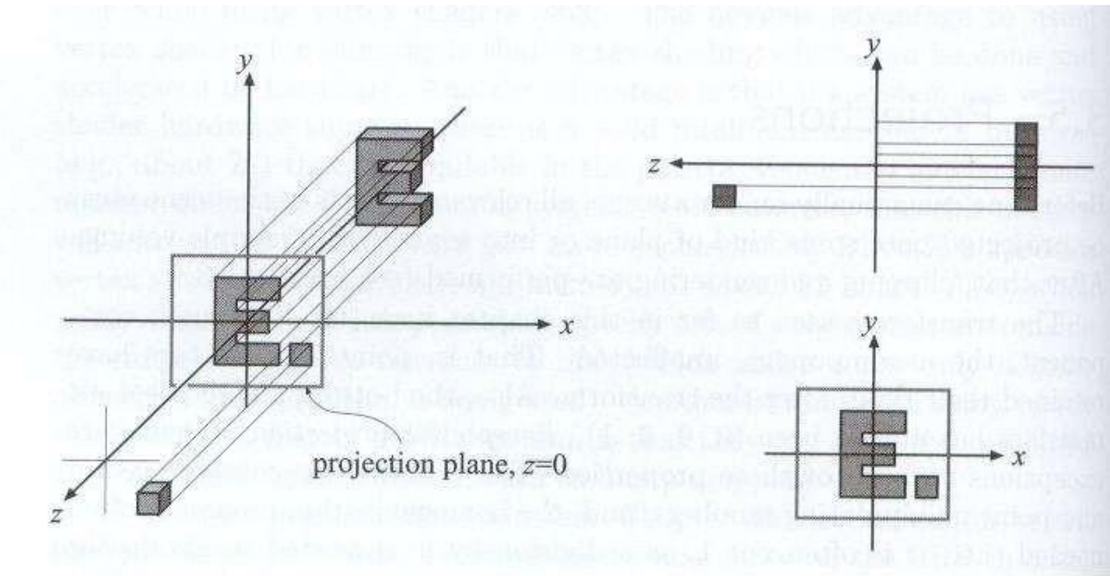
- Perspective projection vs. orthographic projection



https://stackoverflow.com/questions/36573283/from-perspective-picture-to-orthographic-picture

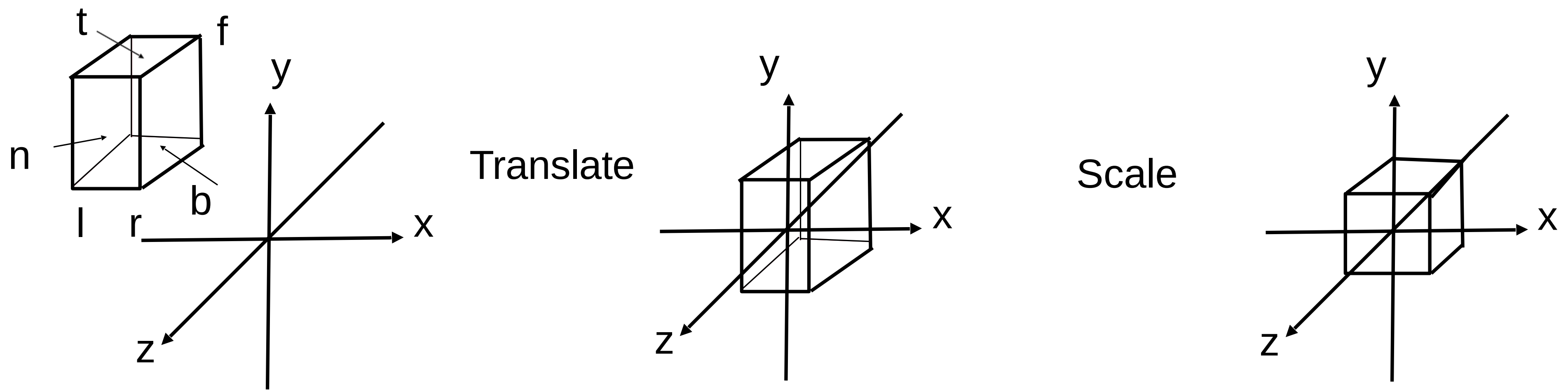
Orthographic Projection

- A simple way of understanding
  - Camera located at origin, looking at -Z, up at Y (looks familiar?)
  - Drop Z coordinate
  - Translate and scale the resulting rectangle to $[-1,1]^2$



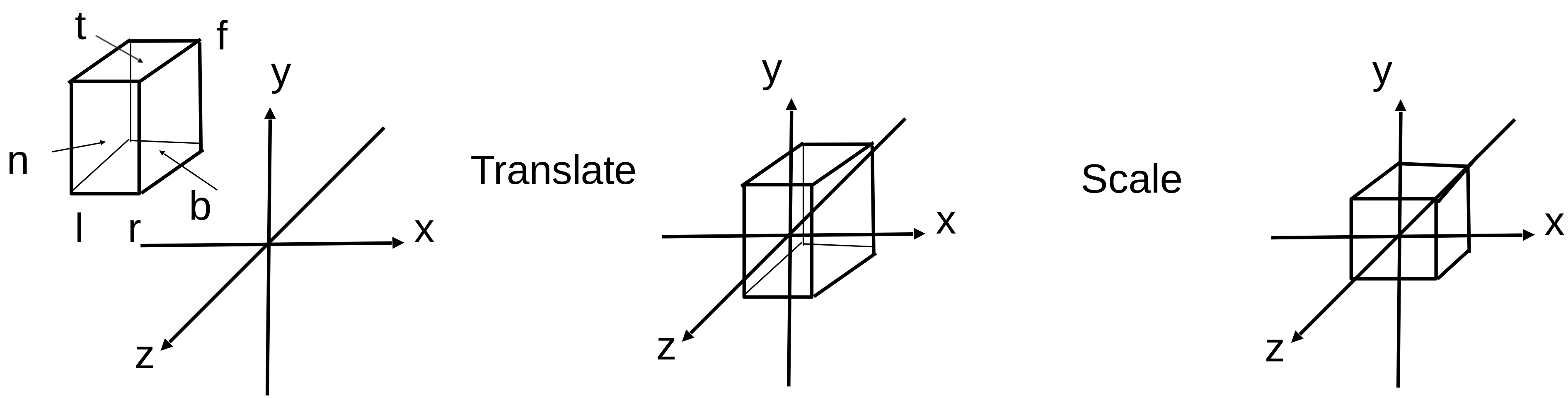
Orthographic Projection

- In general
  - We want to map a cuboid [l, r] x [b, t] x [f, n] to the “canonical (正则、规范、标准)” cube $[-1, 1]^3$



Orthographic Projection

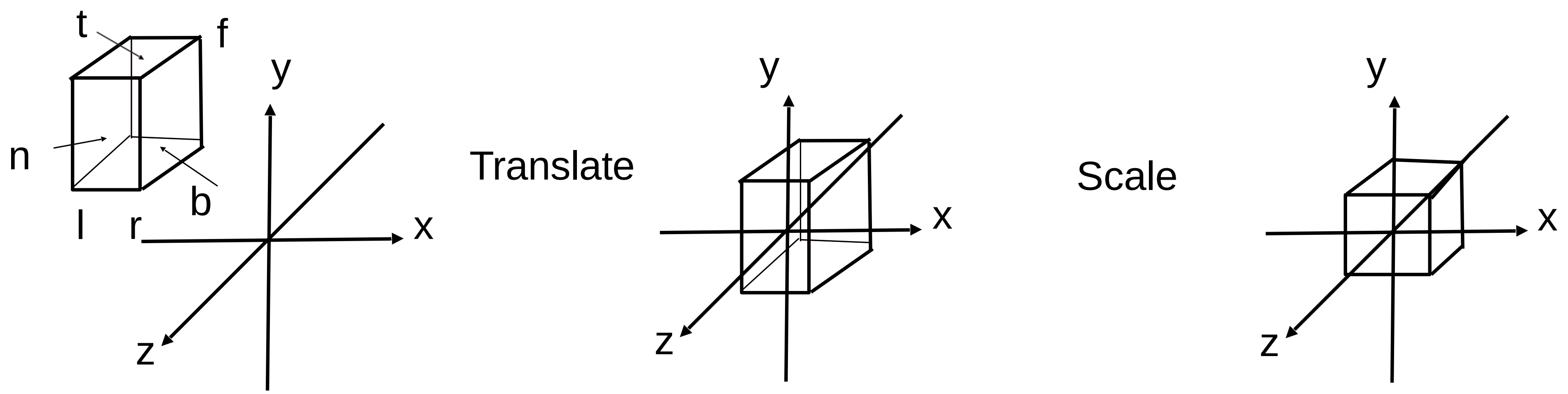
- Slightly different orders (to the "simple way")
  - Center cuboid by translating
  - Scale into "canonical" cube



Orthographic Projection

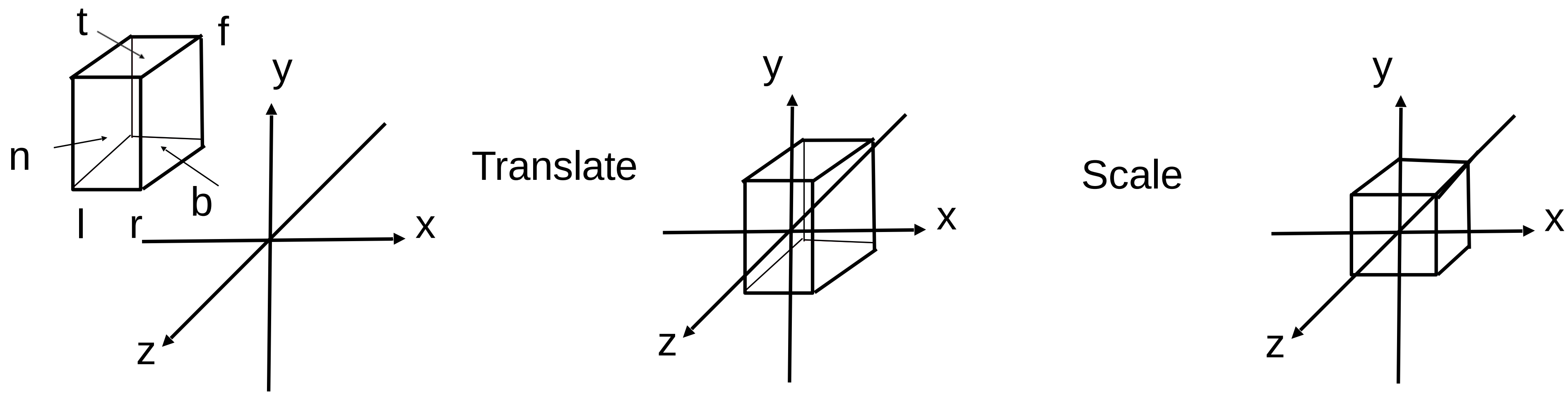
- Transformation matrix?
  - Translate (center to origin) first, then scale (length/width/height to 2)

$$M_{ortho} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Orthographic Projection

- Caveat
  - Looking at / along -Z is making near and far not intuitive (n > f)
  - FYI: that's why OpenGL (a Graphics API) uses left hand coords.



Thank you!

Last Lecture

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  - Homogeneous coordinates
  - Composite transform
  - 3D transformations

- Viewing transformation
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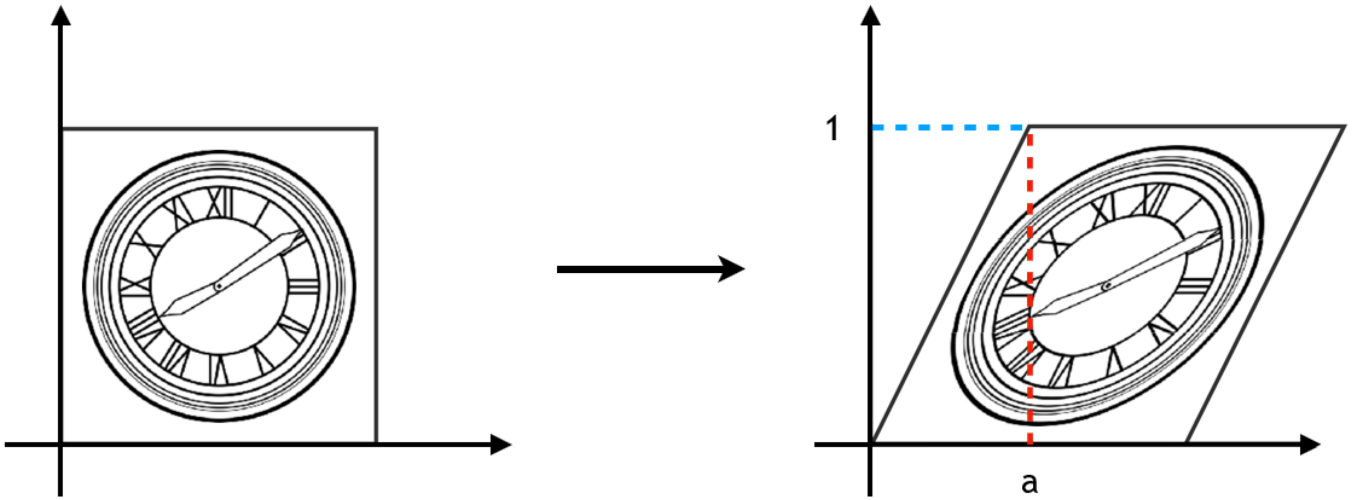
填空题 4分 设置

请填写A, B, C, D处对应的值分别是 [填空1] [填空2] [填空3] [填空4] 。

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{C} & \mathrm{D} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



作答

多选题 1分

设置

如下变换，属于线性变换的有？

A Scale Transformation

B Shear Transformation

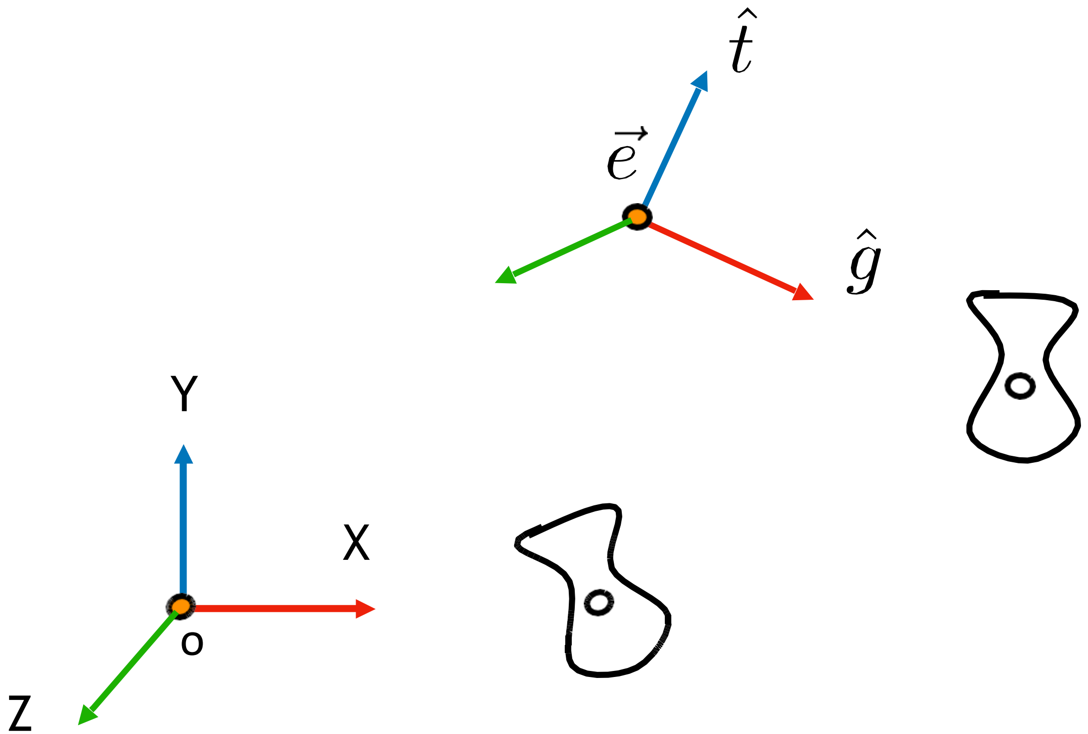
C Translation Transformation

D Rotate Transformation

E Reflection Transformation

提交

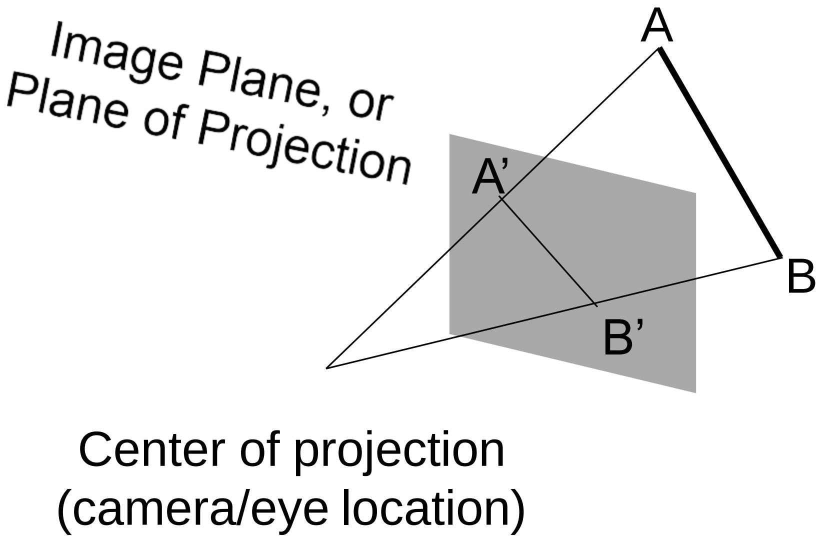
模型视图变换中，相机的位置e变换到 [填空1] ， g变换到 [填空2] ,t 变换到 [填空3]



作答

Perspective Projection

- Most common in Computer Graphics, art, visual system
- Further objects are smaller
- Parallel lines not parallel; converge to single point



Perspective Projection

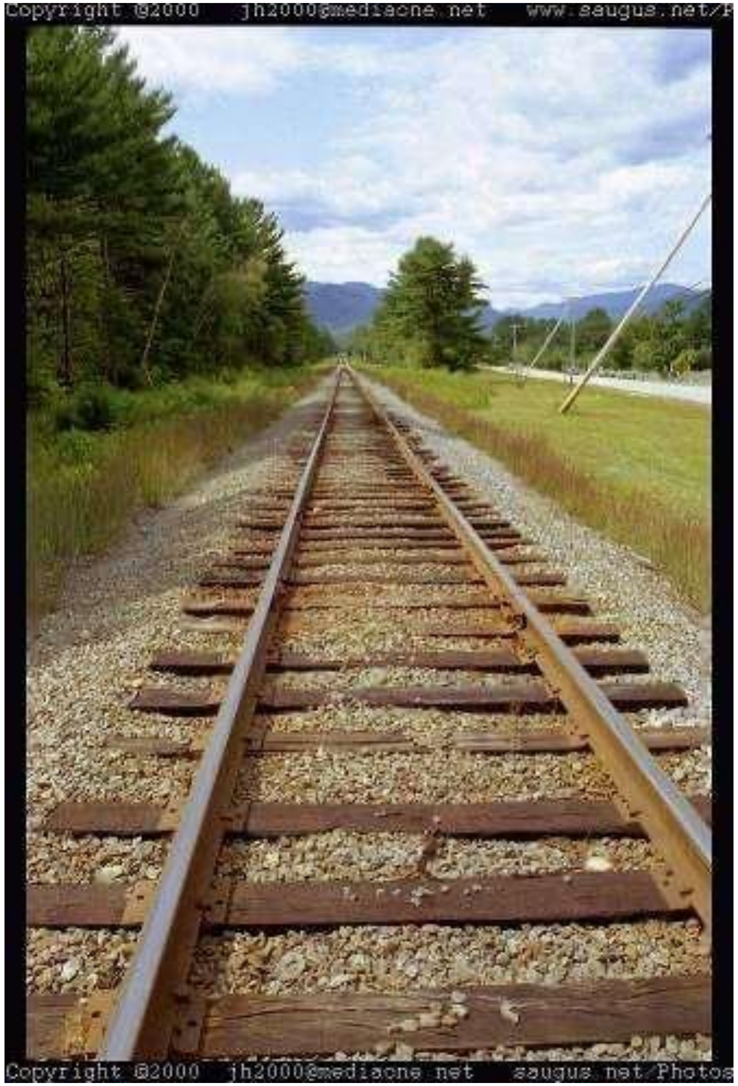
- Euclid was wrong??!!

In geometry, **parallel** lines are lines in a plane which do not meet; that is, two lines in a plane that do not intersect or touch each other at any point are said to be parallel. By extension, a line and a plane, or two planes, in three-dimensional Euclidean space that do not share a point are said to be parallel. However, two lines in three-dimensional space which do not meet must be in a common plane to be considered parallel; otherwise they are called skew lines. Parallel planes are planes in the same three-dimensional space that never meet.

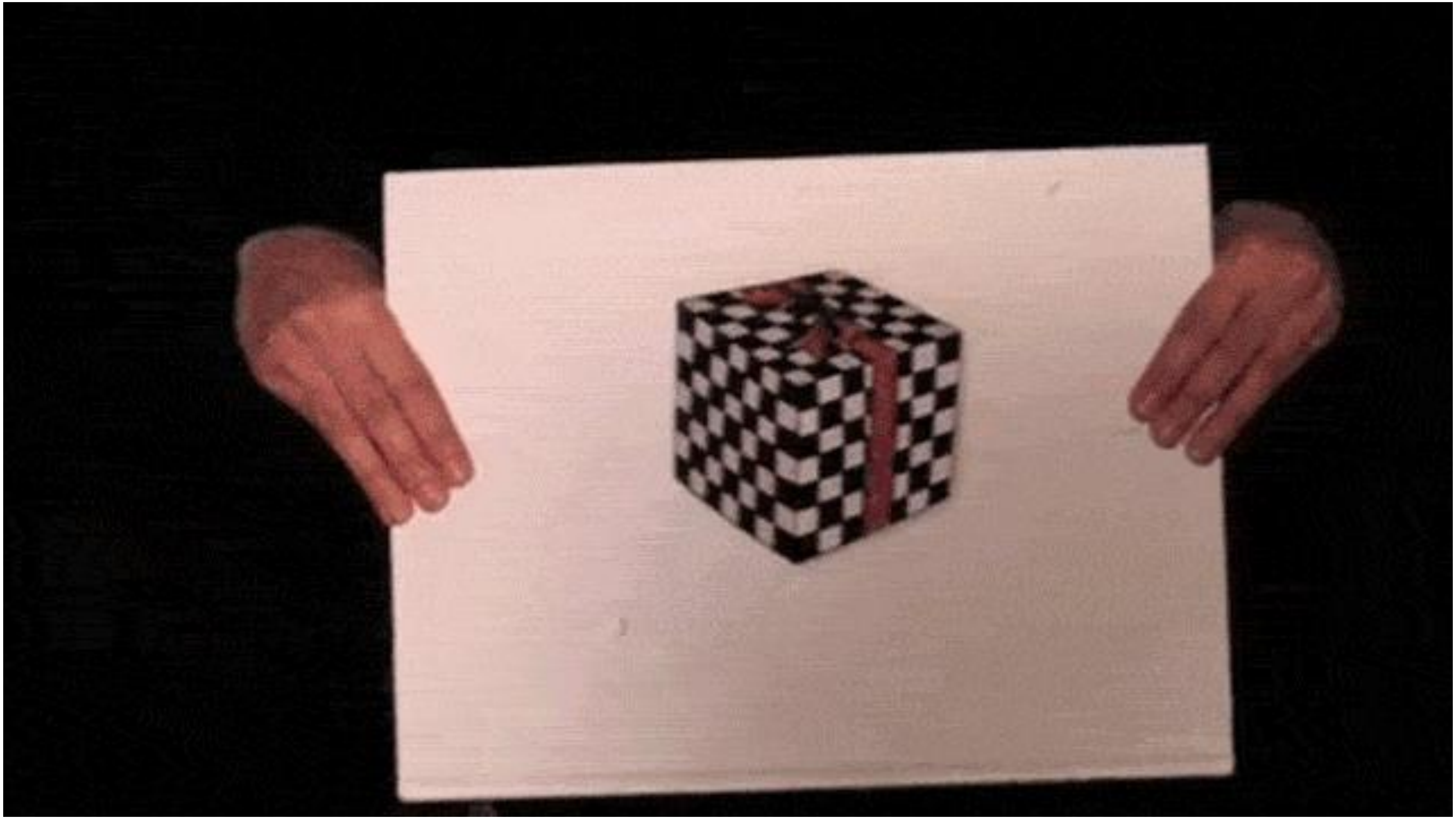
Line art drawing of parallel lines and curves.

Parallel lines are the subject of Euclid's parallel postulate.[1] Parallelism is primarily a property of affine geometries and Euclidean geometry is a special instance of this type of geometry. In some other geometries, such as hyperbolic geometry, lines can have analogous properties that are referred to as parallelism.

https://en.wikipedia.org/wiki/Parallel_(geometry)



Perspective Projection




GATHERING FOR
GARDNER

Perspective Projection

- Before we move on
- Recall: property of homogeneous coordinates
  - $(x, y, z, 1)$, $(kx, ky, kz, k \neq 0)$, $(xz, yz, z^2, z \neq 0)$ all represent the same point (x, y, z) in 3D
  - e.g. $(1, 0, 0, 1)$ and $(2, 0, 0, 2)$ both represent $(1, 0, 0)$
- Simple, but useful

Perspective Projection

- How to do perspective projection
  - First "squish" the frustum into a cuboid (n -> n, f -> f) ($M_{persp->ortho}$)
  - Do orthographic projection (M_{ortho}, already known!)

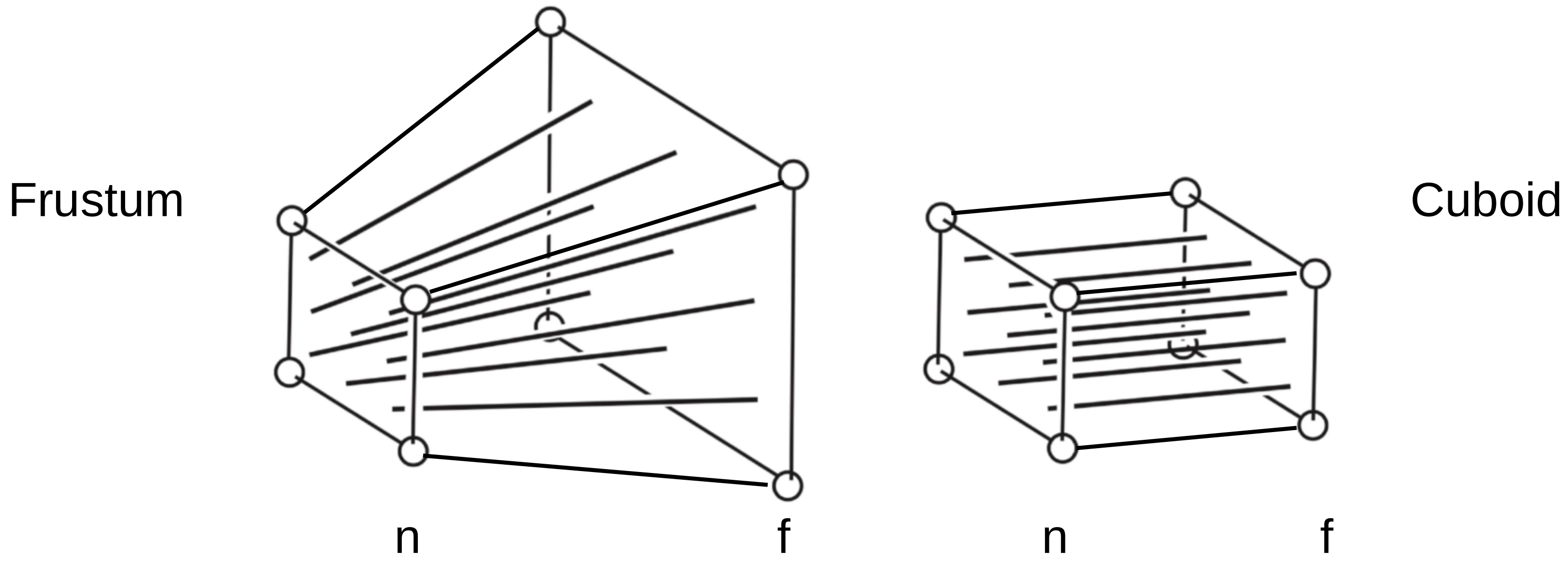
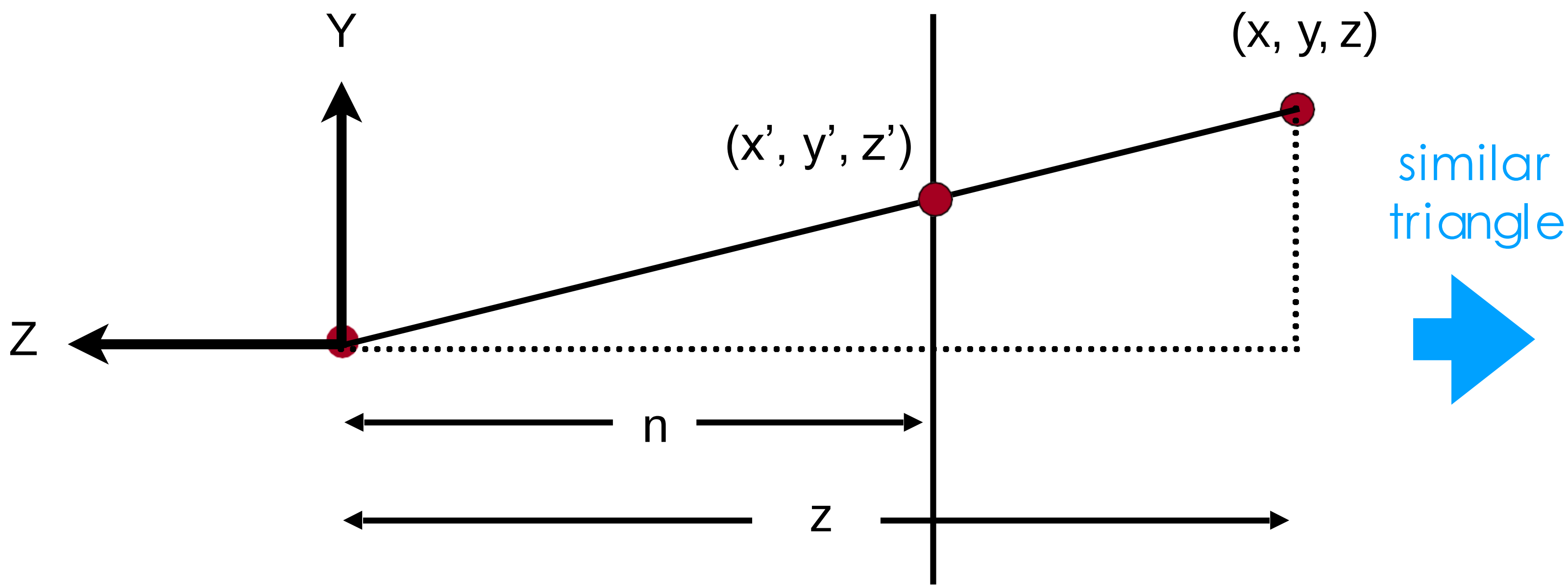


Fig. 7.13 from *Fundamentals of Computer Graphics, 4th Edition*

Perspective Projection

- In order to find a transformation
  - Recall the key idea: Find the relationship between transformed points (x', y', z') and the original points (x, y,z)



$$y' = \frac{n}{z} y$$

Perspective Projection

- In order to find a transformation
  - Find the relationship between transformed points (x′, y′, z′) and the original points (x, y, z)

$$y' = \frac{n}{z} y \qquad x' = \frac{n}{z} x$$ (similar to y')

- In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} nx/z \\ ny/z \\ \text{unknown} \\ 1 \end{pmatrix} \overset{\substack{\text{mult.} \\ \text{by z}}}{==} \begin{pmatrix} nx \\ ny \\ \text{still unknown} \\ z \end{pmatrix}$$

Perspective Projection

- So the "squish" (persp to ortho) projection does this

$$M_{persp\to ortho}^{(4\times 4)} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ \text{unknown} \\ z \end{pmatrix}$$

- Already good enough to figure out part of $M_{persp\text{->}ortho}$

$$M_{persp\to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

WHY?

Perspective Projection

- How to figure out the third row of $M_{persp\text{->}ortho}$
  - Any information that we can use?

$$M_{persp \to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- Observation: the third row is responsible for z'
  - Any point on the near plane will not change
  - Any point's z on the far plane will not change

Perspective Projection

- Any point on the near plane will not change

$$M_{persp \to ortho}^{(4\times4)} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ \text{unknown} \\ z \end{pmatrix}$$

replace
z with n

$$\begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} == \begin{pmatrix} nx \\ ny \\ n^2 \\ n \end{pmatrix}$$

- So the third row must be of the form (0 0 A B)

$$\begin{pmatrix} 0 & 0 & A & B \end{pmatrix} \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = n^2$$

n^2 has nothing to do with x and y

Perspective Projection

- What do we have now?

$$\begin{pmatrix} 0 & 0 & A & B \end{pmatrix} \begin{pmatrix} x \\ y \\ n \\ 1 \end{pmatrix} = n^2 \quad \Rightarrow \quad An + B = n^2$$

- Any point's z on the far plane will not change

$$\begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ f \\ 1 \end{pmatrix} == \begin{pmatrix} 0 \\ 0 \\ f^2 \\ f \end{pmatrix} \quad \Rightarrow \quad Af + B = f^2$$

Perspective Projection

- Solve for A and B

$$An + B = n^2 \qquad\qquad A = n + f$$
$$Af + B = f^2 \qquad\qquad B = -nf$$

- Finally, every entry in $M_{persp\text{->}ortho}$ is known!

- What's next?
  - Do orthographic projection (M_{ortho}) to finish
  - $M_{persp} = M_{ortho} M_{persp \rightarrow ortho}$

Thank you!