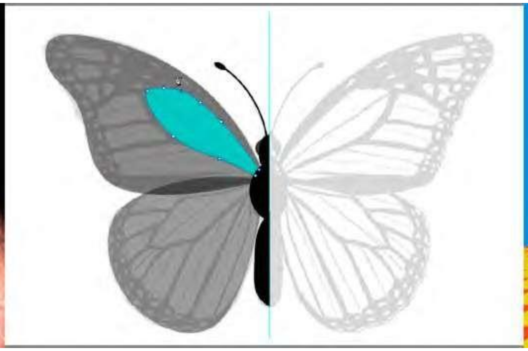
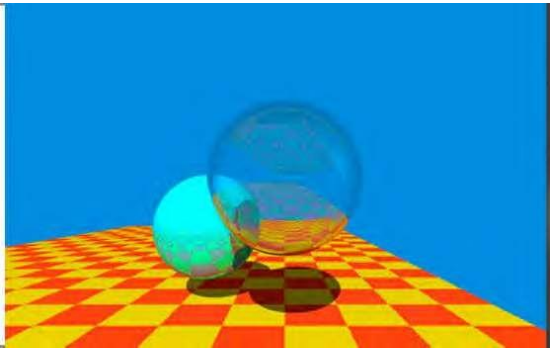
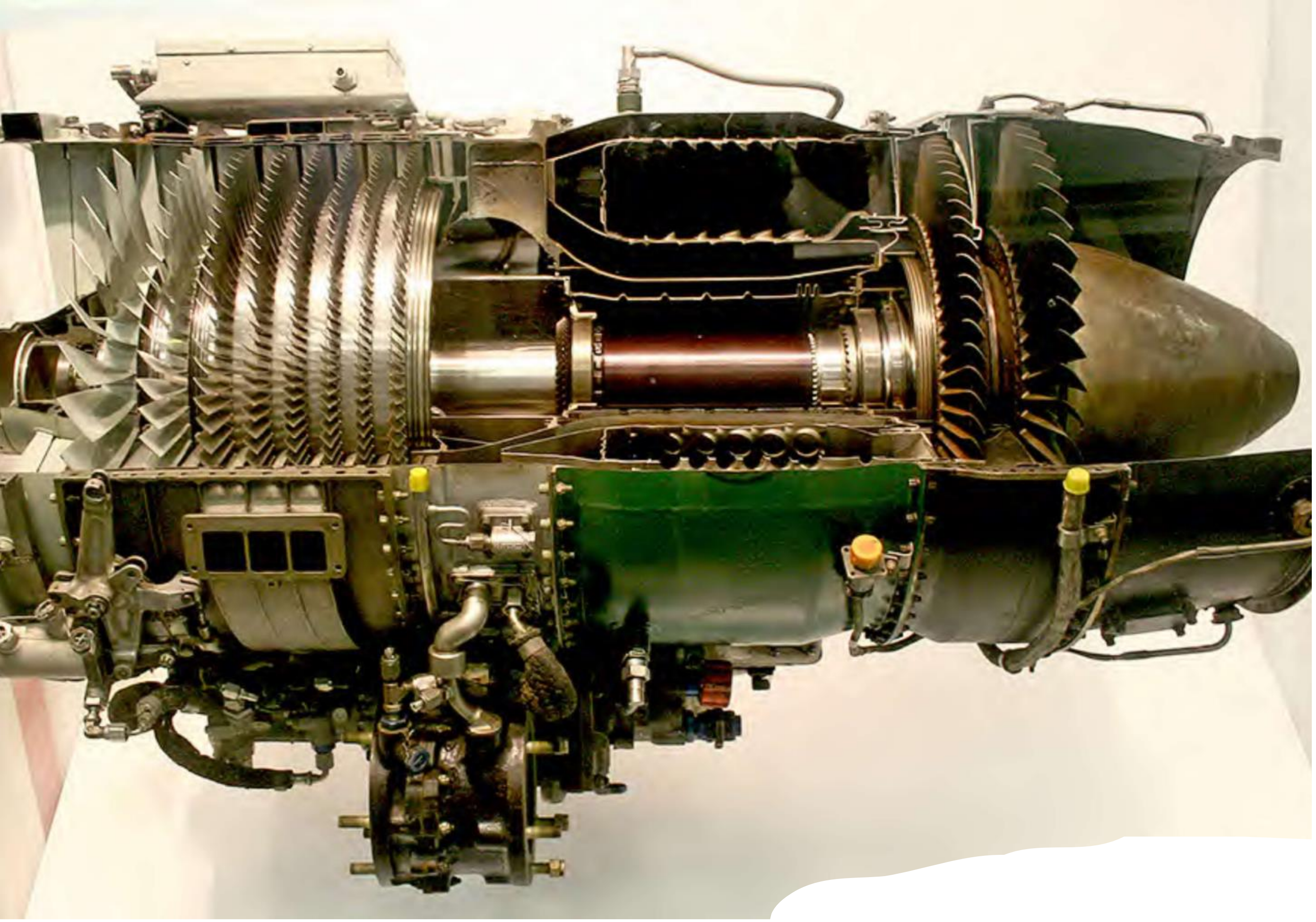
Computer Graphics

Geometry 1 (Introduction)


Today

- Introduction to geometry
  - Examples of geometry
  - Various representations of geometry



Examples of Geometry

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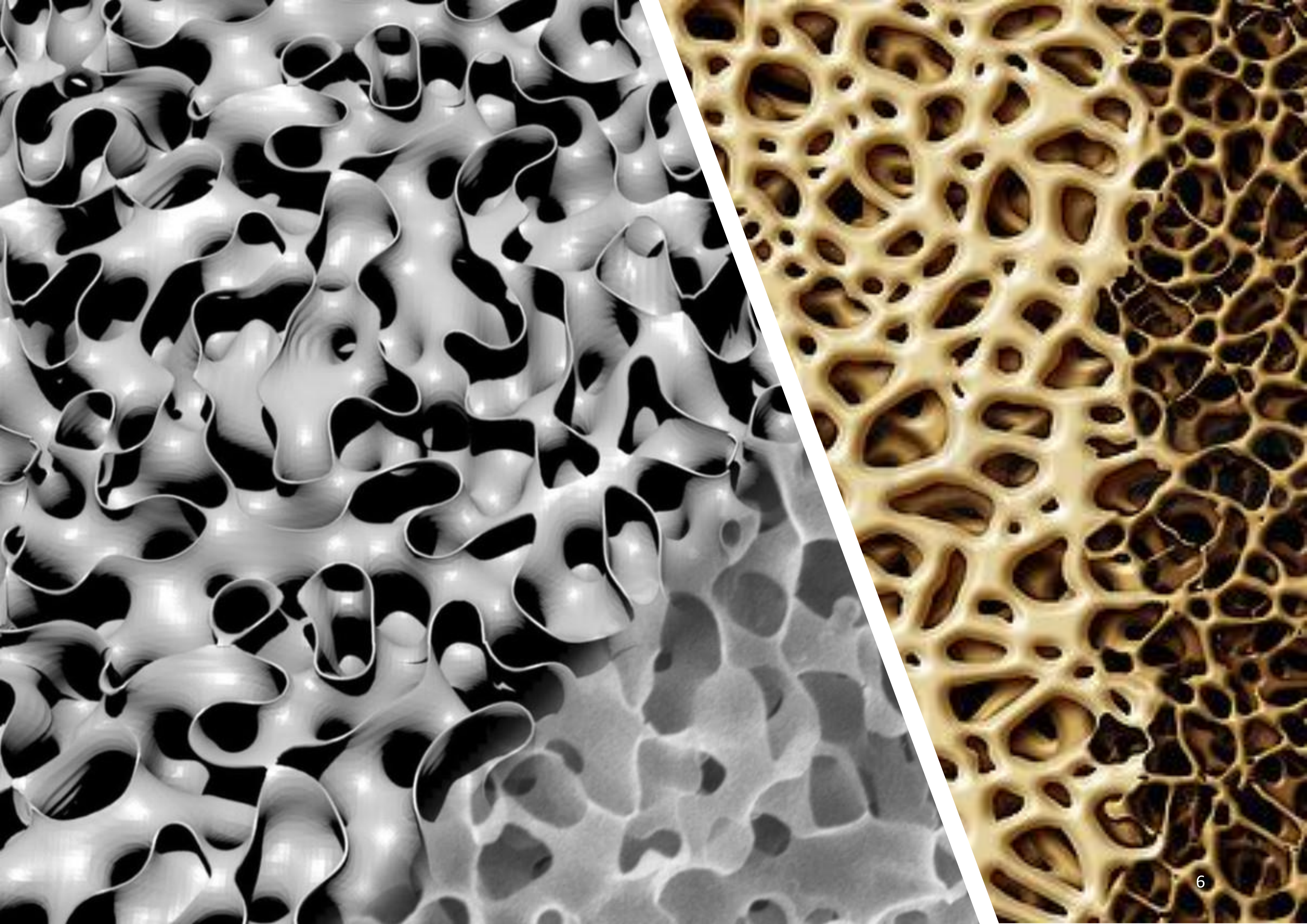
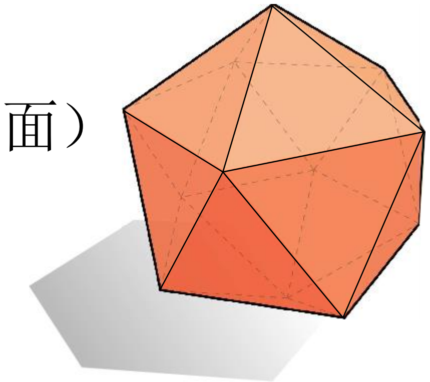
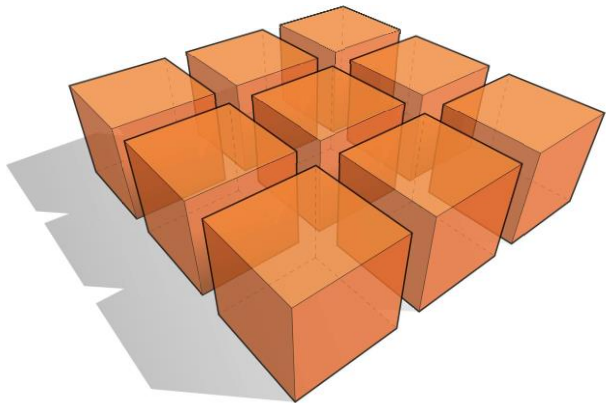


Many Ways to Represent Geometry

Implicit

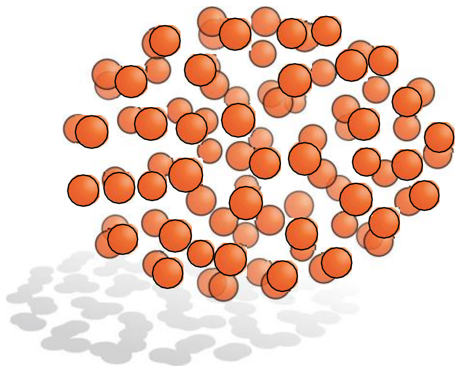
- algebraic surface （代数曲面）
- level sets （水平集）
- distance functions
- ...

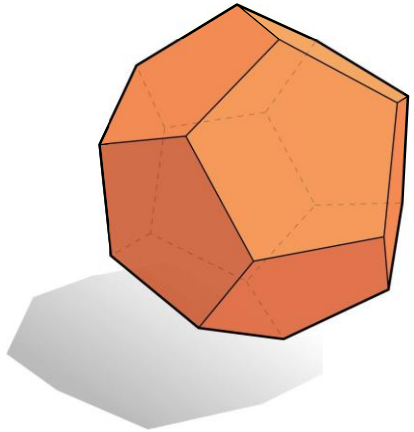


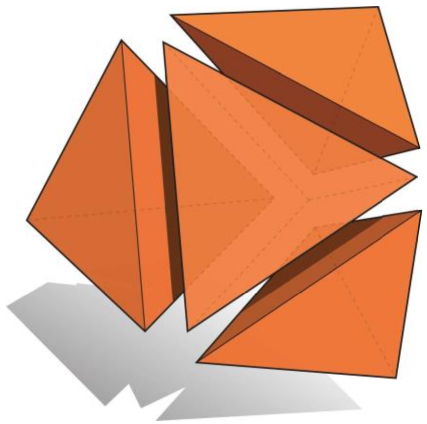


Explicit

- point cloud
- polygon mesh
- subdivision, NURBS
- ...







Each choice best suited to a different task/type of geometry

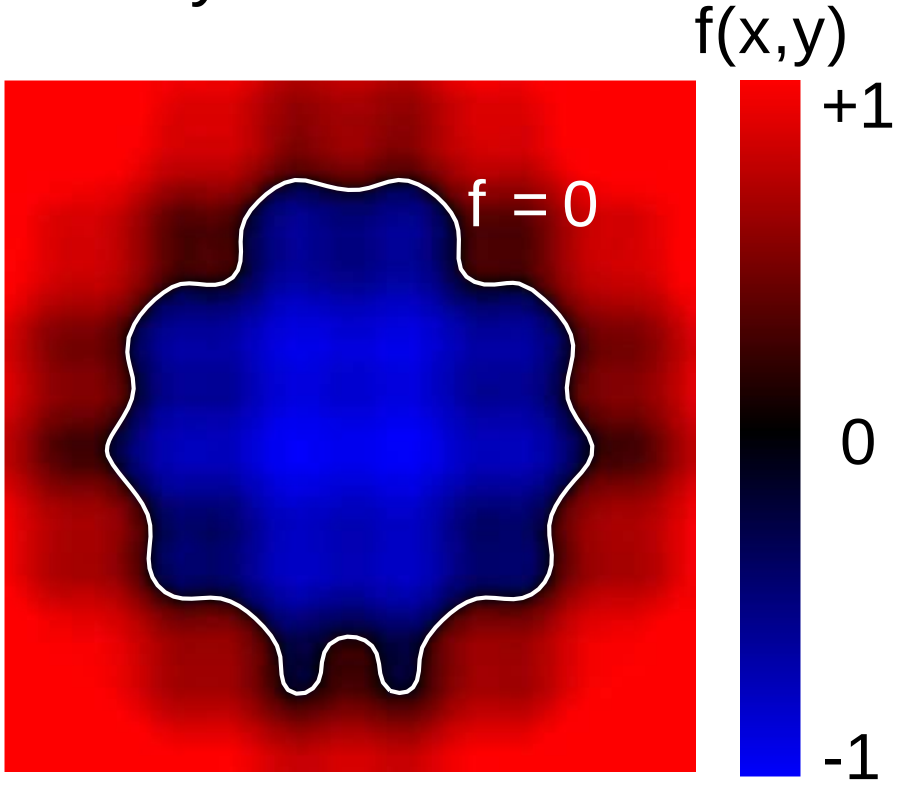
"Implicit" Representations of Geometry

Based on classifying points

- Points satisfy some specified relationship

E.g. sphere: all points in 3D, where $x^2+y^2+z^2 = 1$

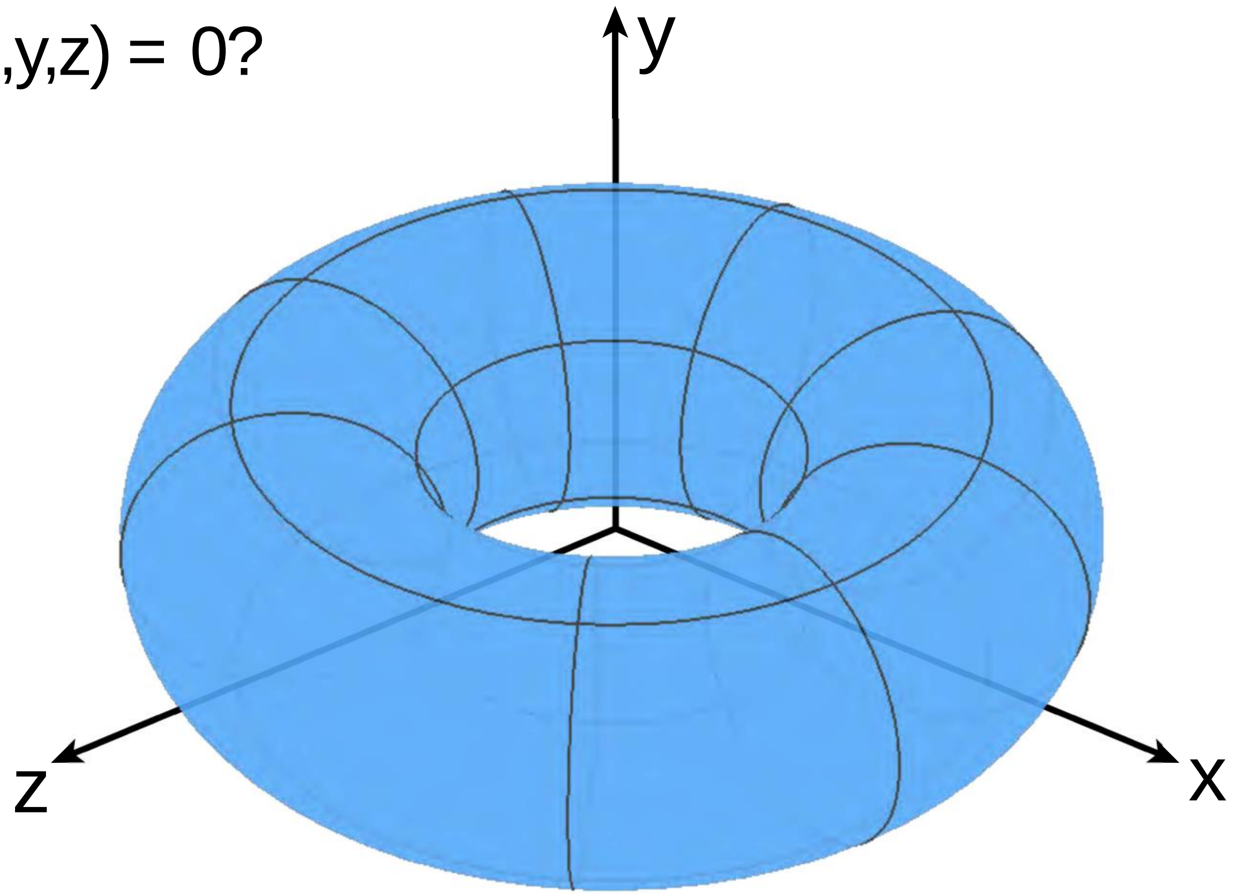
More generally, $f(x,y,z) = 0$



Implicit Surface – Sampling Can Be Hard

$$f(x,y,z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$$

What points lie on f(x,y,z) = 0?

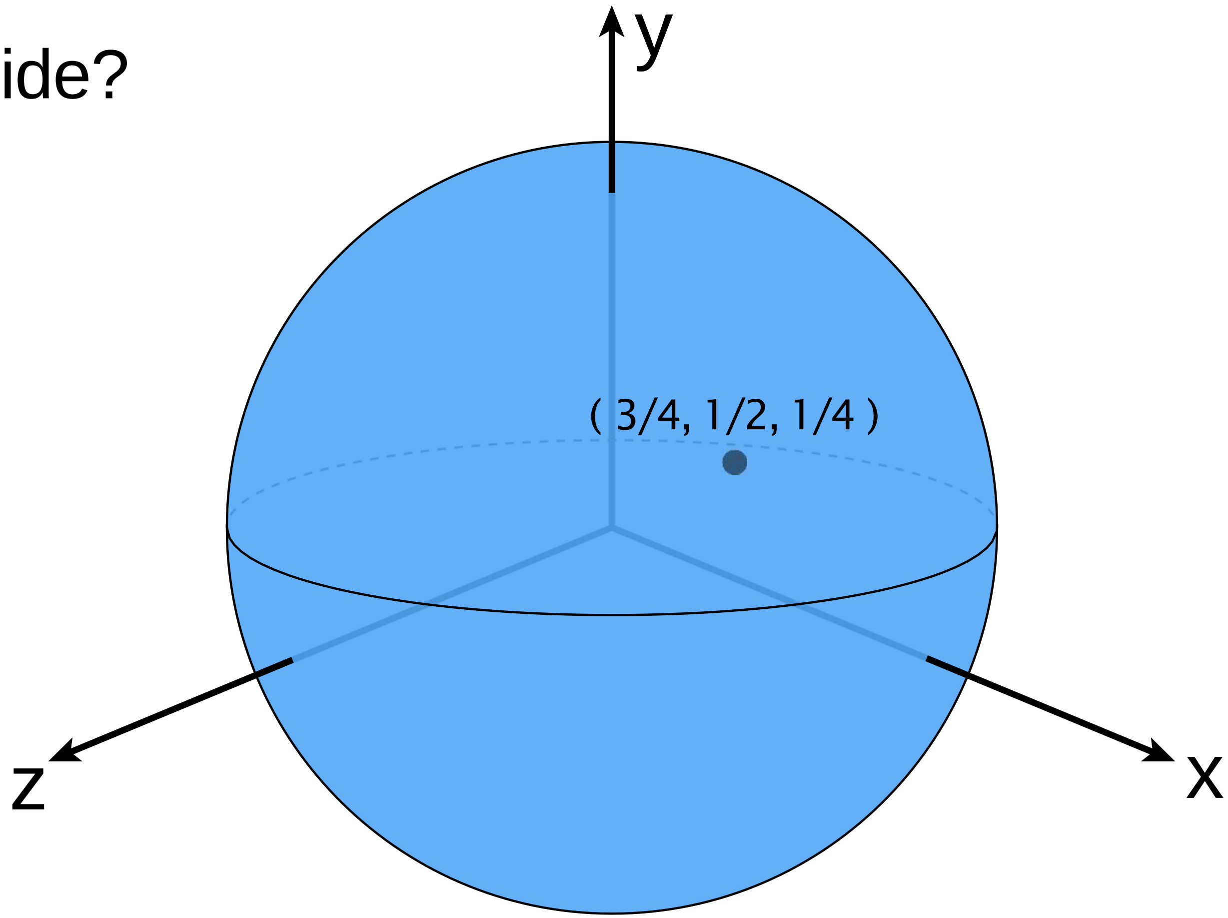


Some tasks are hard with implicit representations

Implicit Surface – Inside/Outside Tests Easy

$$f(x, y, z) = x^2 + y^2 + z^2 - 1$$

Is (3/4, 1/2, 1/4) inside?



Just plug it in:

$f(x,y,z) = -1/8 < 0$

Yes, inside.

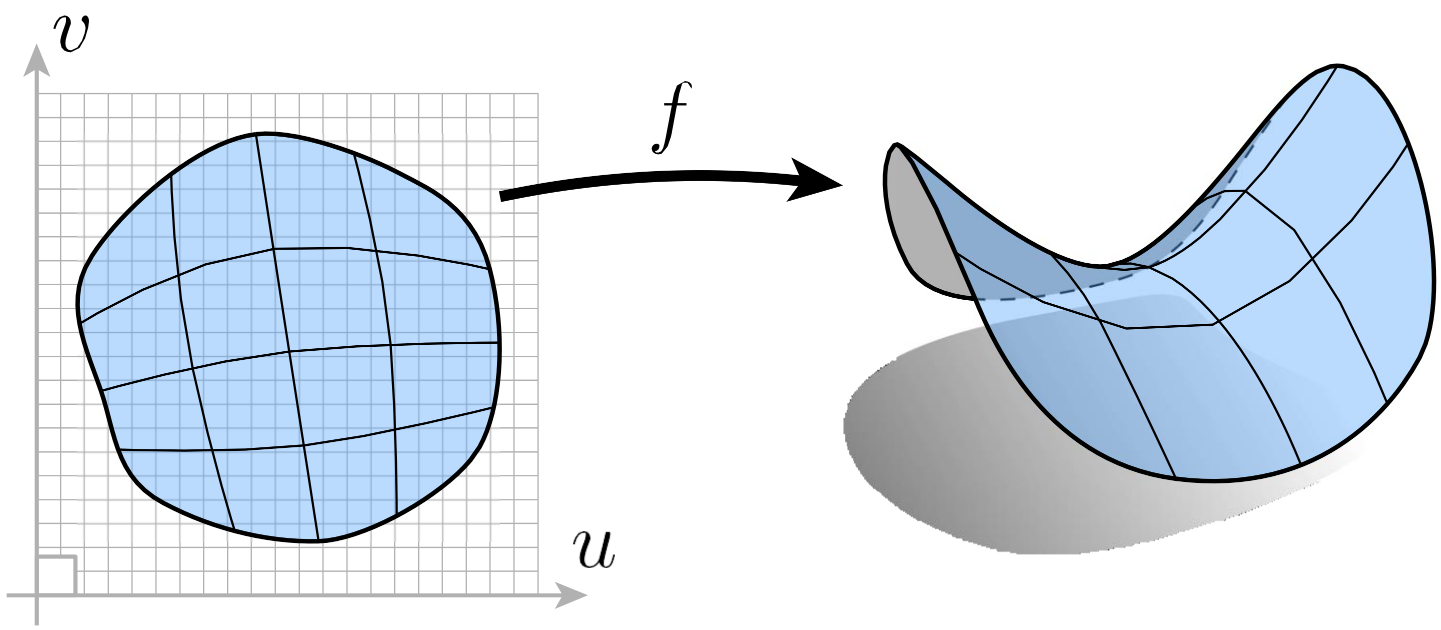
Implicit representations make some tasks easy

"Explicit" Representations of Geometry

All points are given directly or via parameter mapping

Generally:

$$f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$$

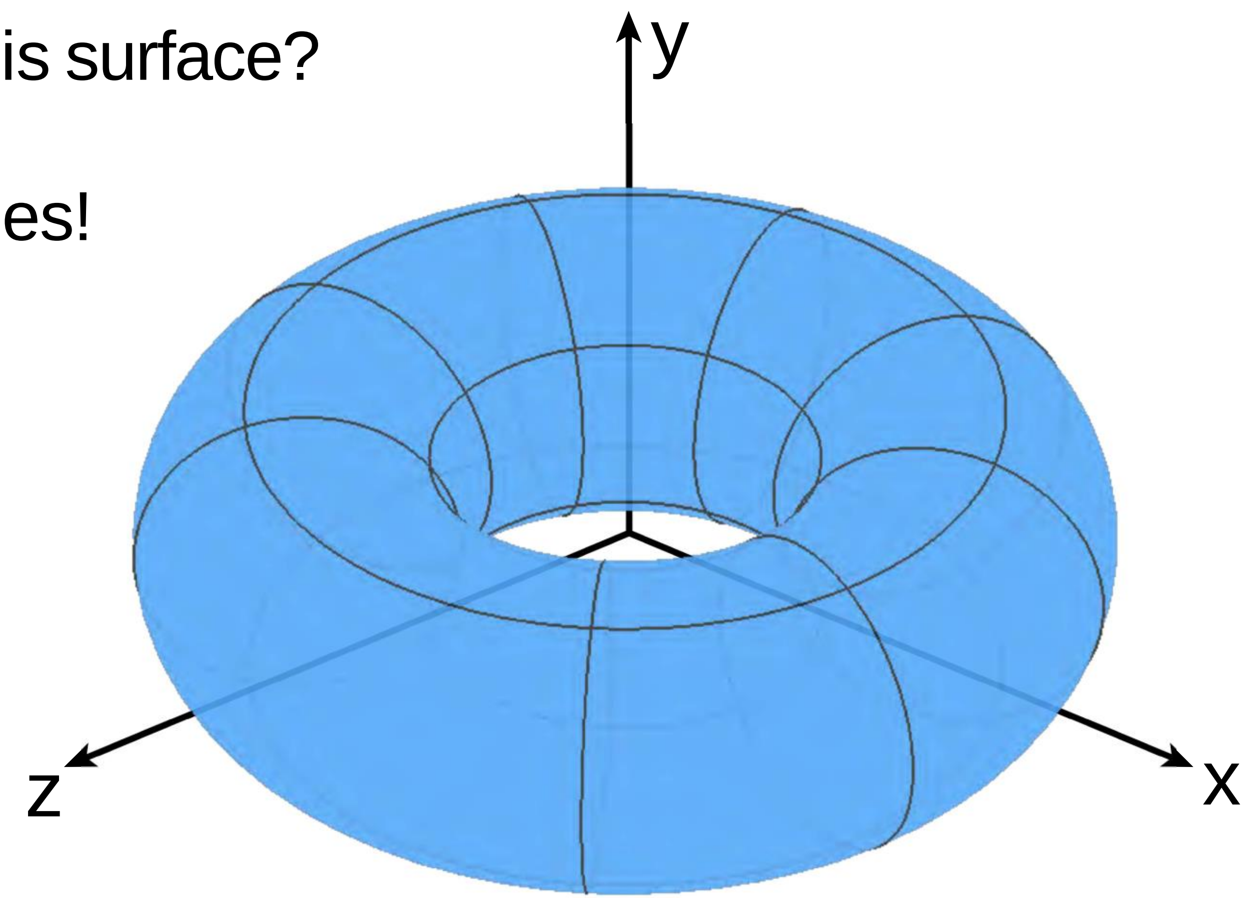


Explicit Surface – Sampling Is Easy

$$f(u,v) = ((2 + \cos u)\cos v, (2 + \cos u)\sin v, \sin u)$$

What points lie on this surface?

Just plug in (u,v) values!

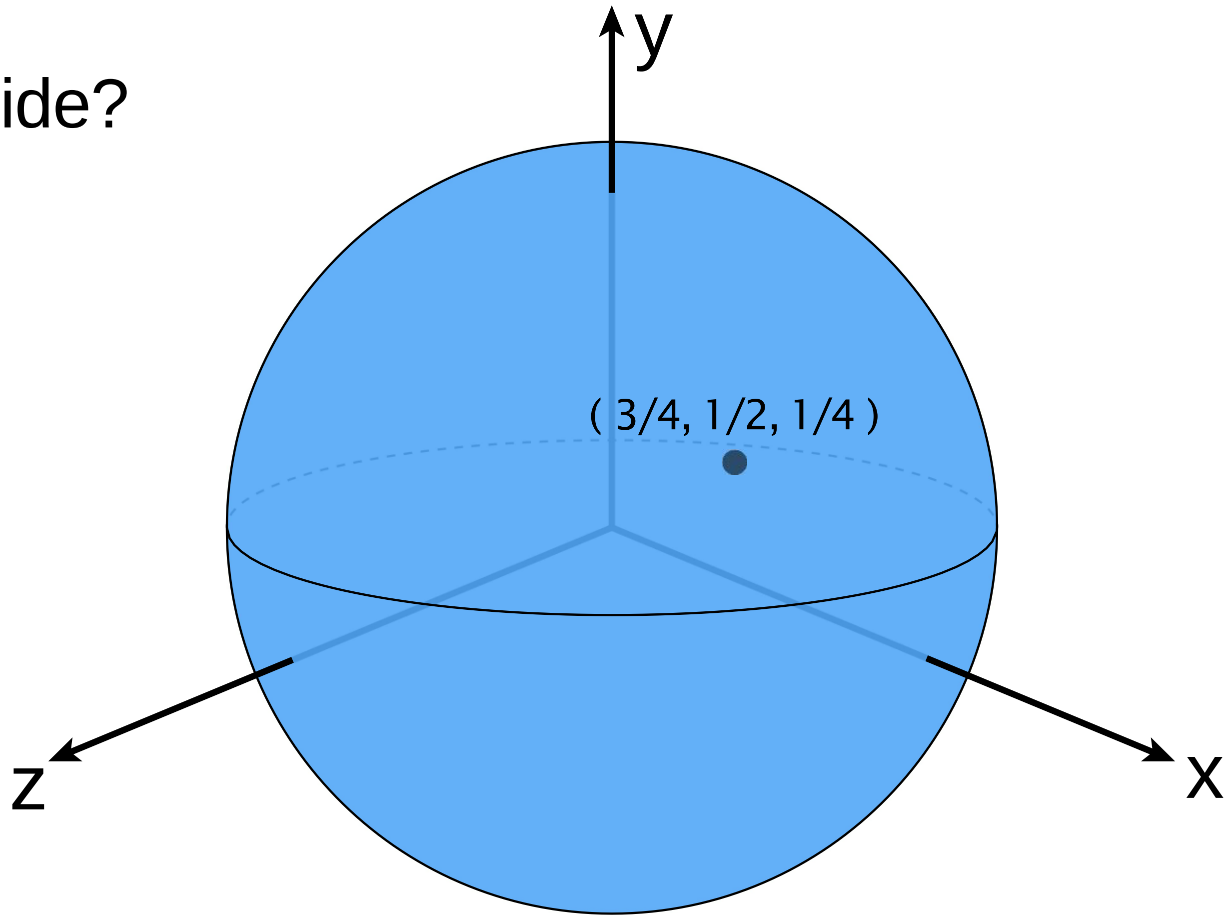


Explicit representations make some tasks easy

Explicit Surface – Inside/Outside Test Hard

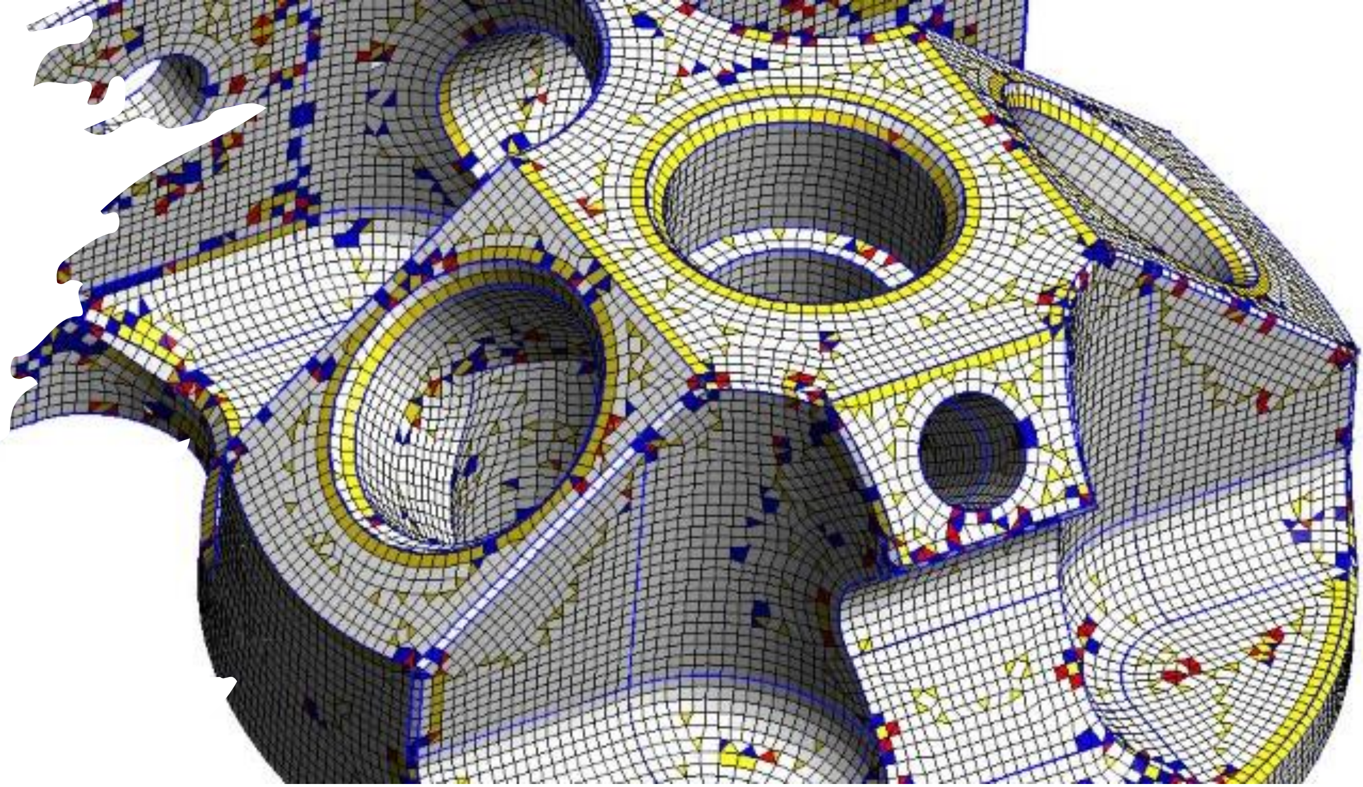
$$f(u, v) = (\cos u \, \sin v, \sin u \, \sin v, \cos v)$$

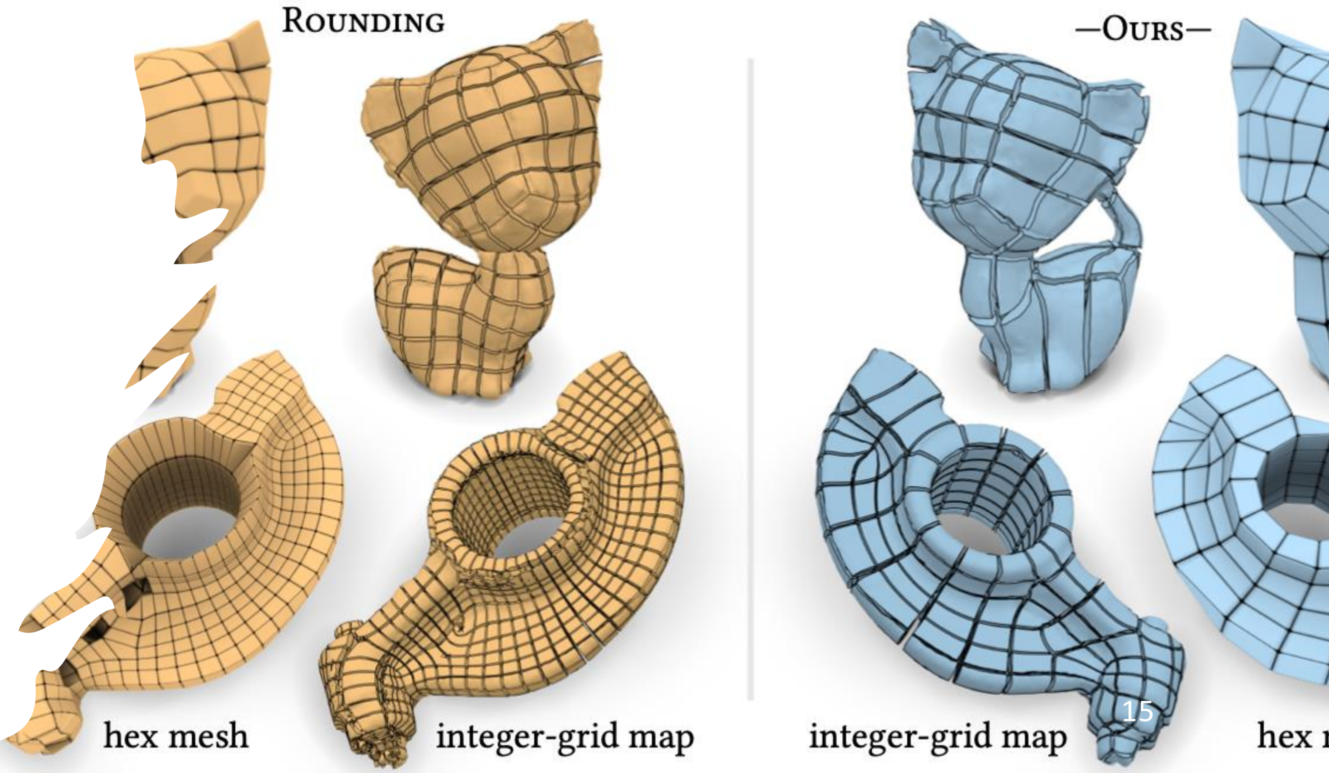
Is (3/4, 1/2, 1/4) inside?



Some tasks are hard with explicit representations

No "Best" Representation – Geometry is Hard!





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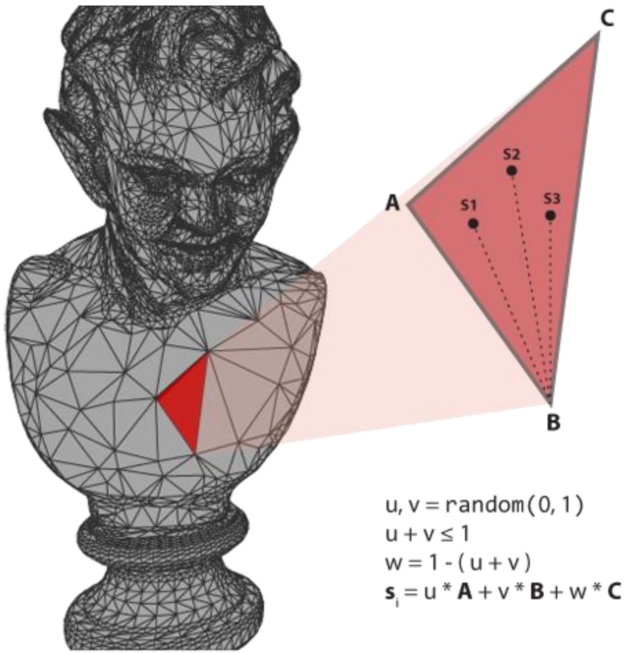
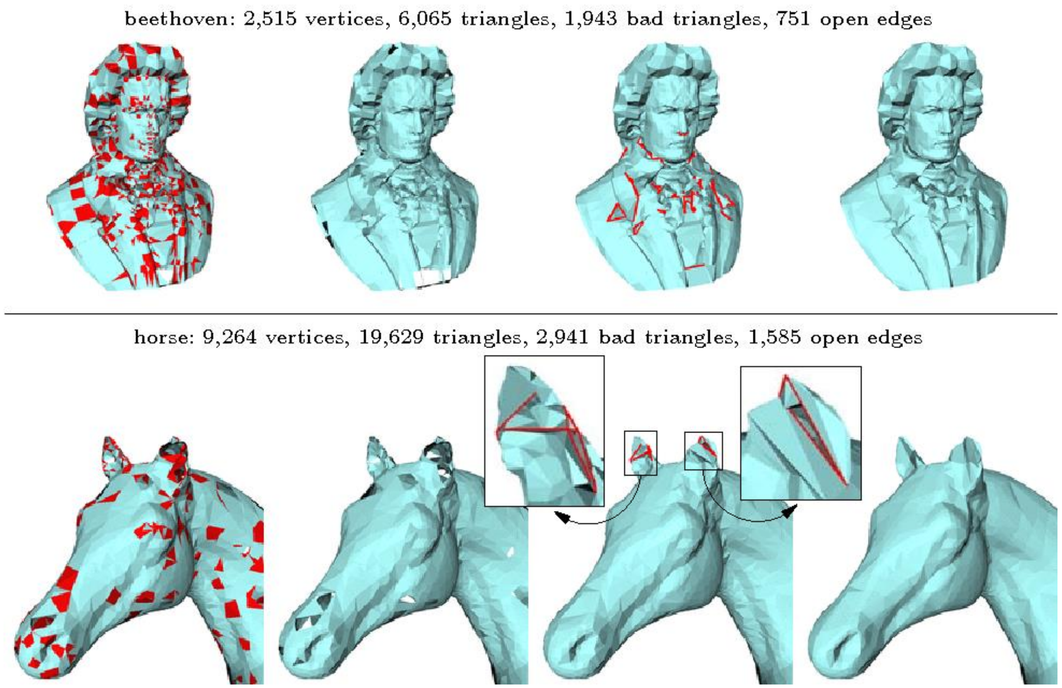
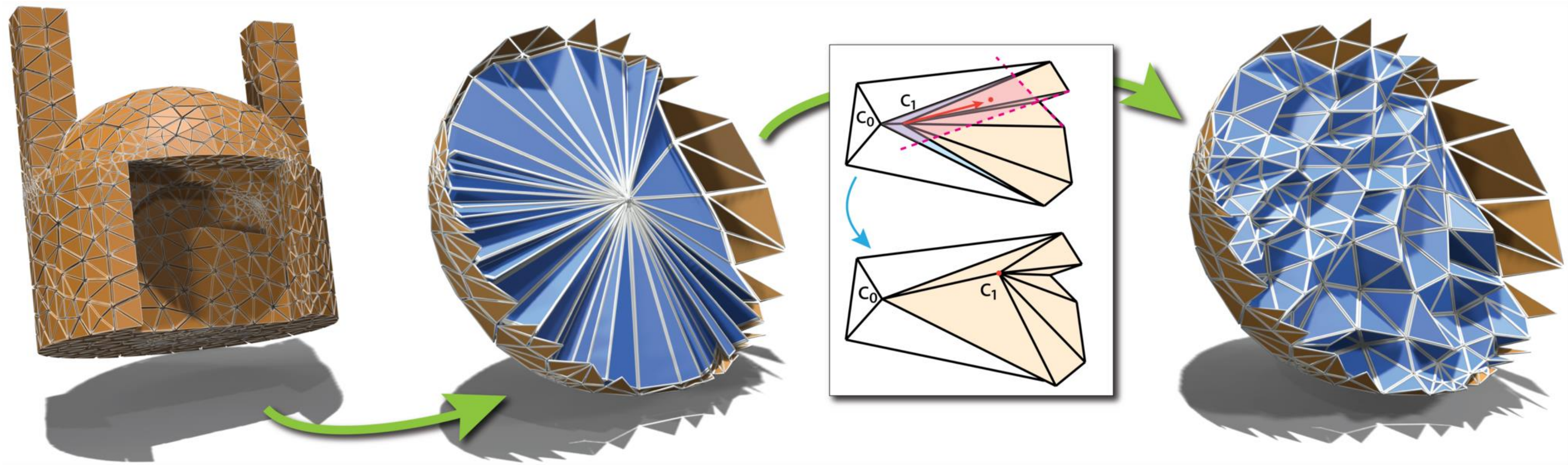






Best Representation Depends on the Task!

More Implicit Representations in Computer Graphics

Many Implicit Representations in Graphics

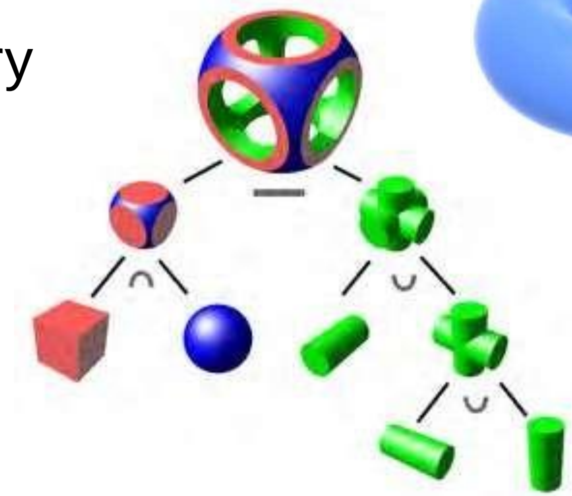
Algebraic surfaces

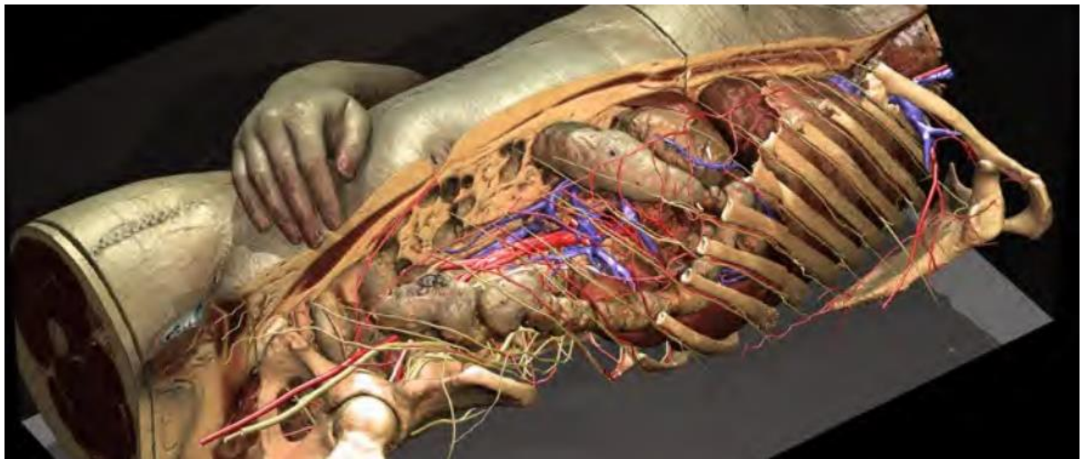
Constructive solid geometry

Level set methods

Fractals （分形）

...





Algebraic Surfaces (Implicit)

Surface is zero set of a polynomial in x, y, z

$$x^2 + y^2 + z^2 = 1$$

$$(R - \sqrt{x^2 + y^2})^2 + z^2 = r^2$$

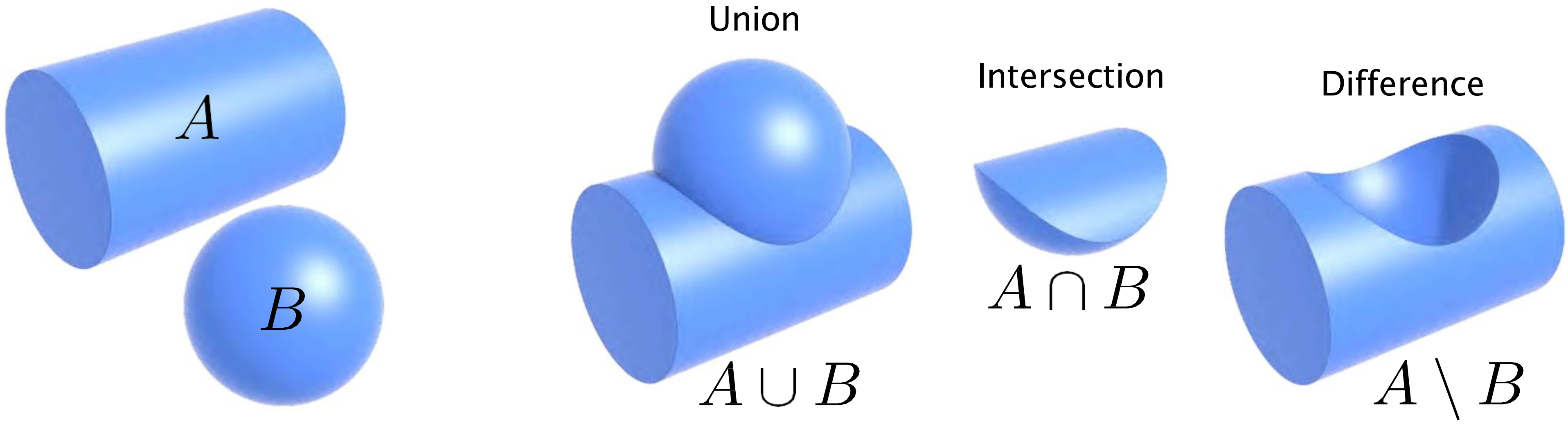
$$(x^2 + \frac{9y^2}{4} + z^2 - 1)^3 = x^2 z^3 + \frac{9y^2 z^3}{80}$$

More complex shapes?

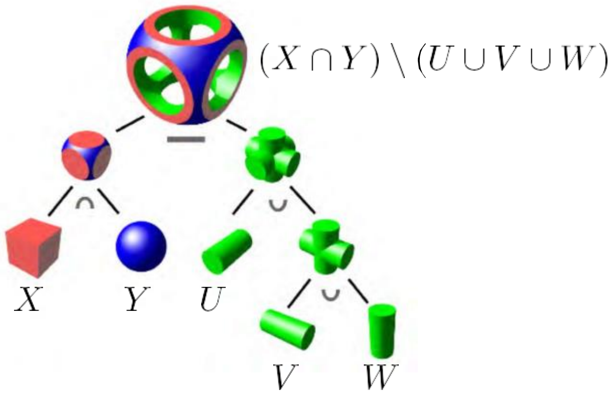
Algebraic Surfaces (Implicit)

Constructive Solid Geometry (Implicit)

Combine implicit geometry via Boolean operations



Boolean expressions:

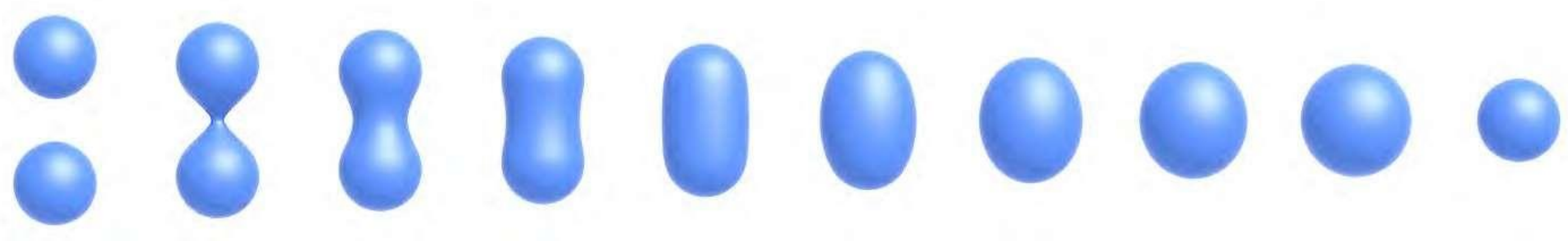


Distance Functions (Implicit)

Instead of Booleans, gradually blend surfaces together using

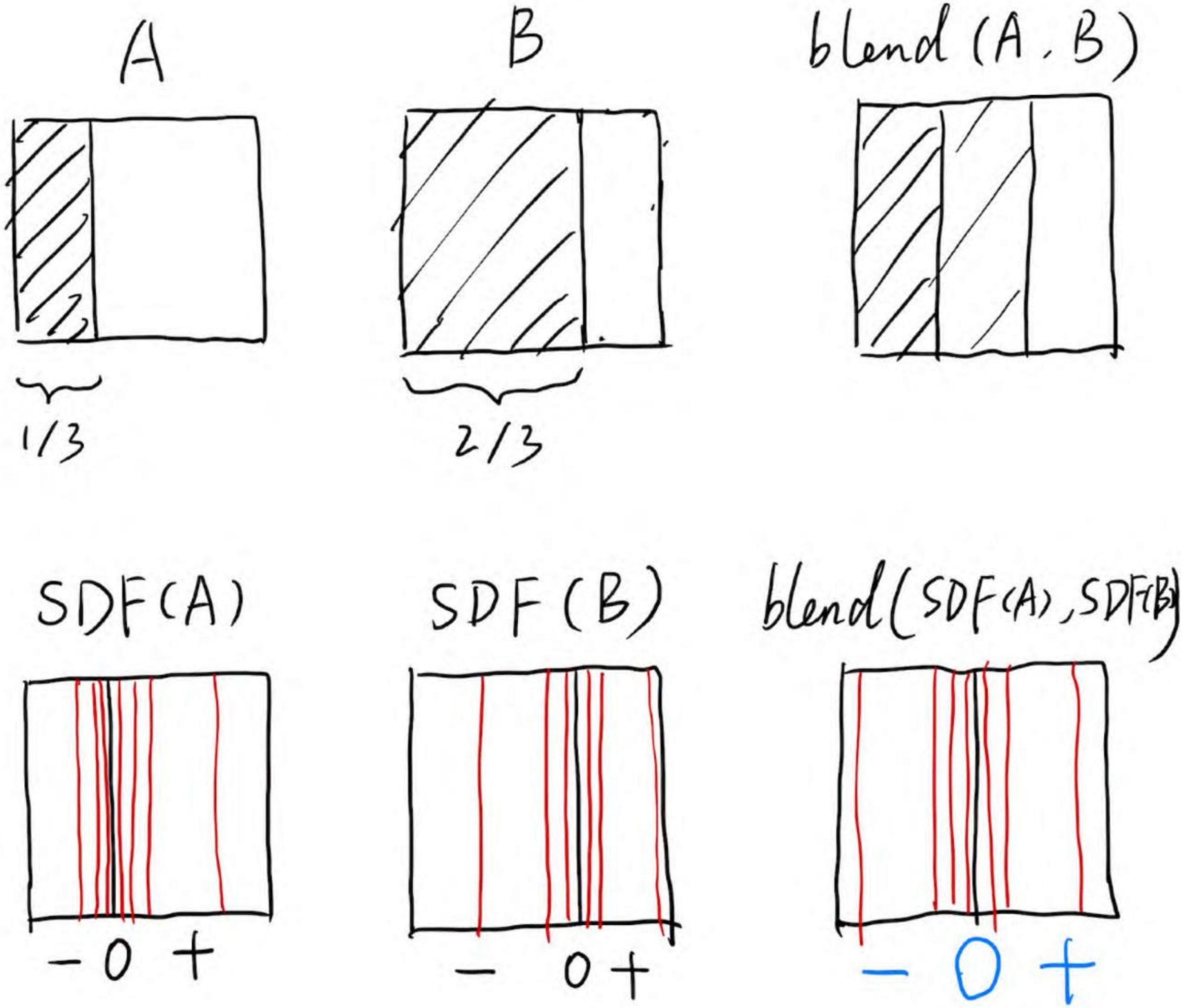
Distance functions:

giving minimum distance (could be signed distance)
from anywhere to object



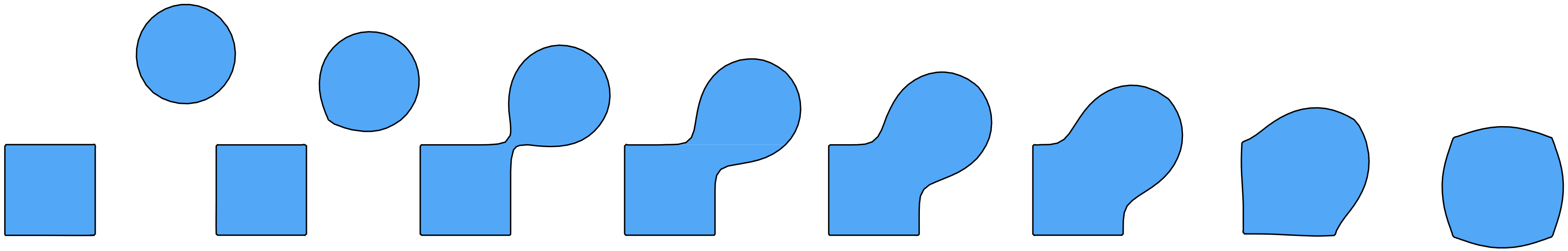
Distance Functions (Implicit)

An Example: Blending (linear interp.) a moving boundary



Blending Distance Functions (Implicit)

Can blend any two distance functions d1, d2:



Scene of Pure Distance Functions

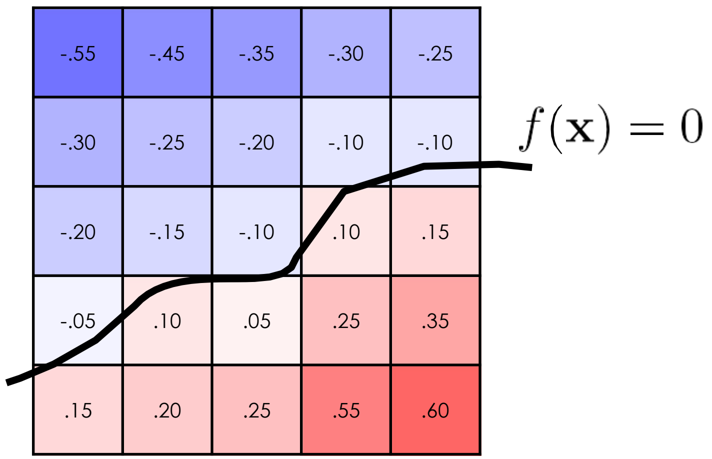
See https://iquilezles.org/www/articles/raymarchingdf/raymarchingdf.htm

Level Set Methods (Also implicit)

（水平集）

Closed-form equations are hard to describe complex shapes

Alternative: store a grid of values approximating function

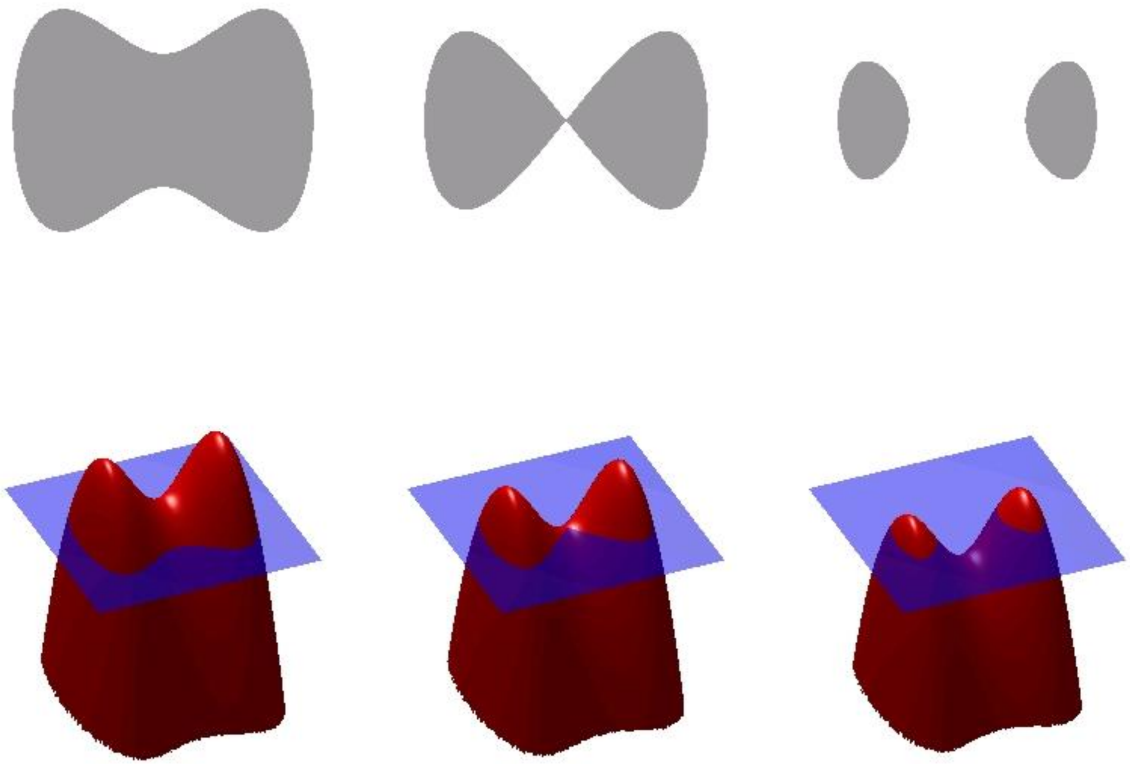


Surface is found where interpolated values equal zero

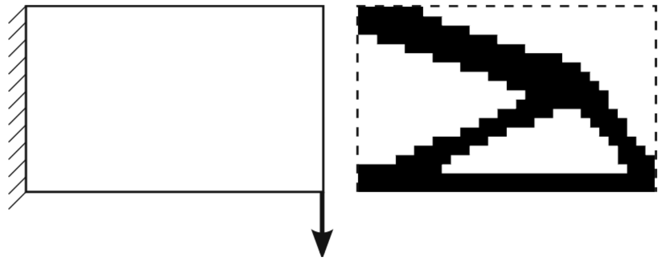
Provides much more explicit control over shape (like a texture)

Level Set Methods (Also implicit)

（水平集）



$$\Gamma = \{(x,y)\,|\,\varphi(x,y)=0\}$$



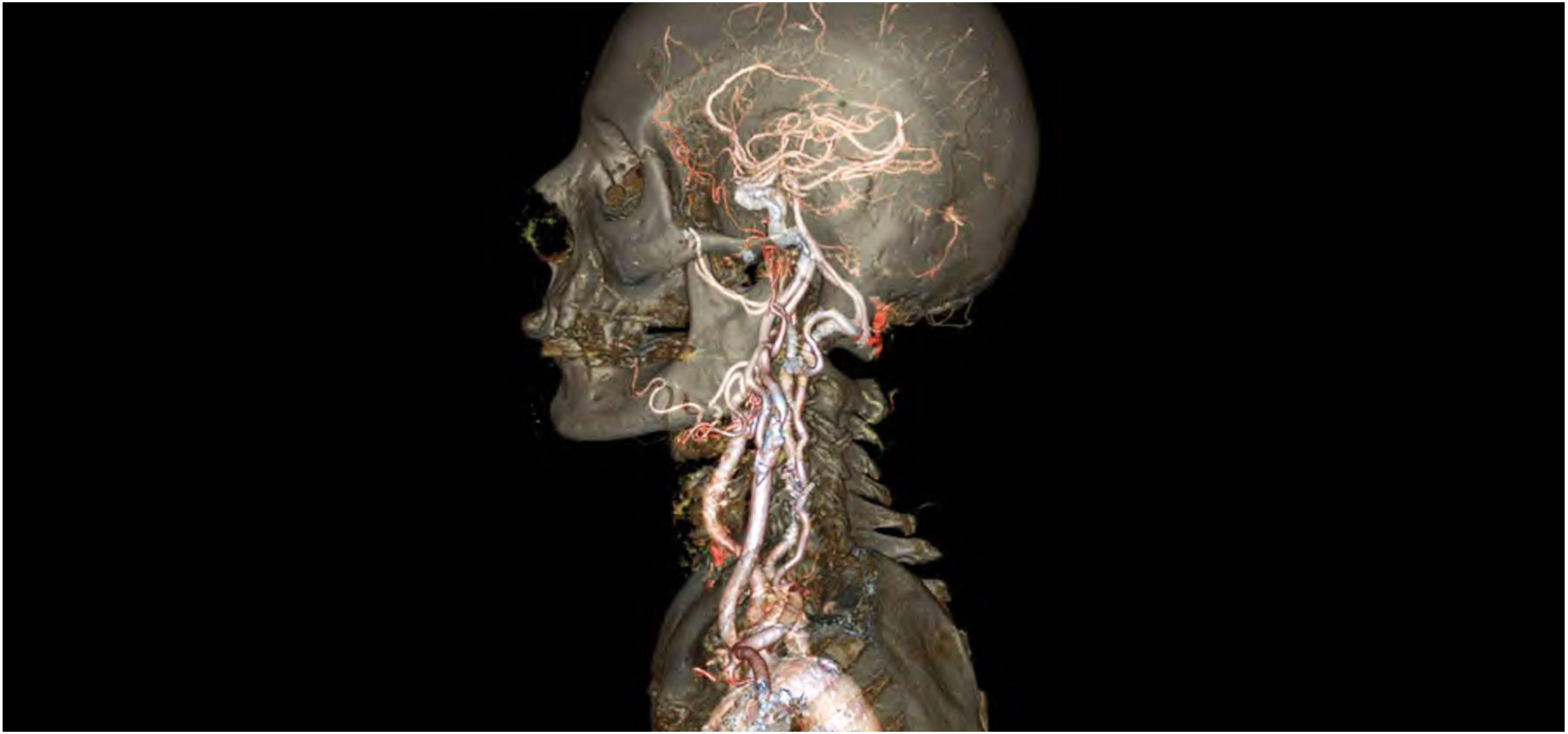
$$\begin{aligned}
\min_{\mathbf{x}} &: c(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} \mathbf{u}_e^{\mathrm{T}}\mathbf{k}_e\mathbf{u}_e = \sum_{e=1}^{N} x_e \mathbf{u}_e^{\mathrm{T}}\mathbf{k}_1\mathbf{u}_e \\
\text{subject to} &: V(\mathbf{x}) = V_{req} \qquad (1) \\
&: \mathbf{K}\mathbf{U} = \mathbf{F} \\
&: x_e = 0 \text{ or } x_e = 1 \quad \forall\, e = 1, \ldots, N
\end{aligned}$$

基于水平集方法的拓扑优化

https://zh.wikipedia.org/wiki/%E6%B0%B4%E5%B9%B3%E9%9B%86%E6%96%B9%E6%B3%95

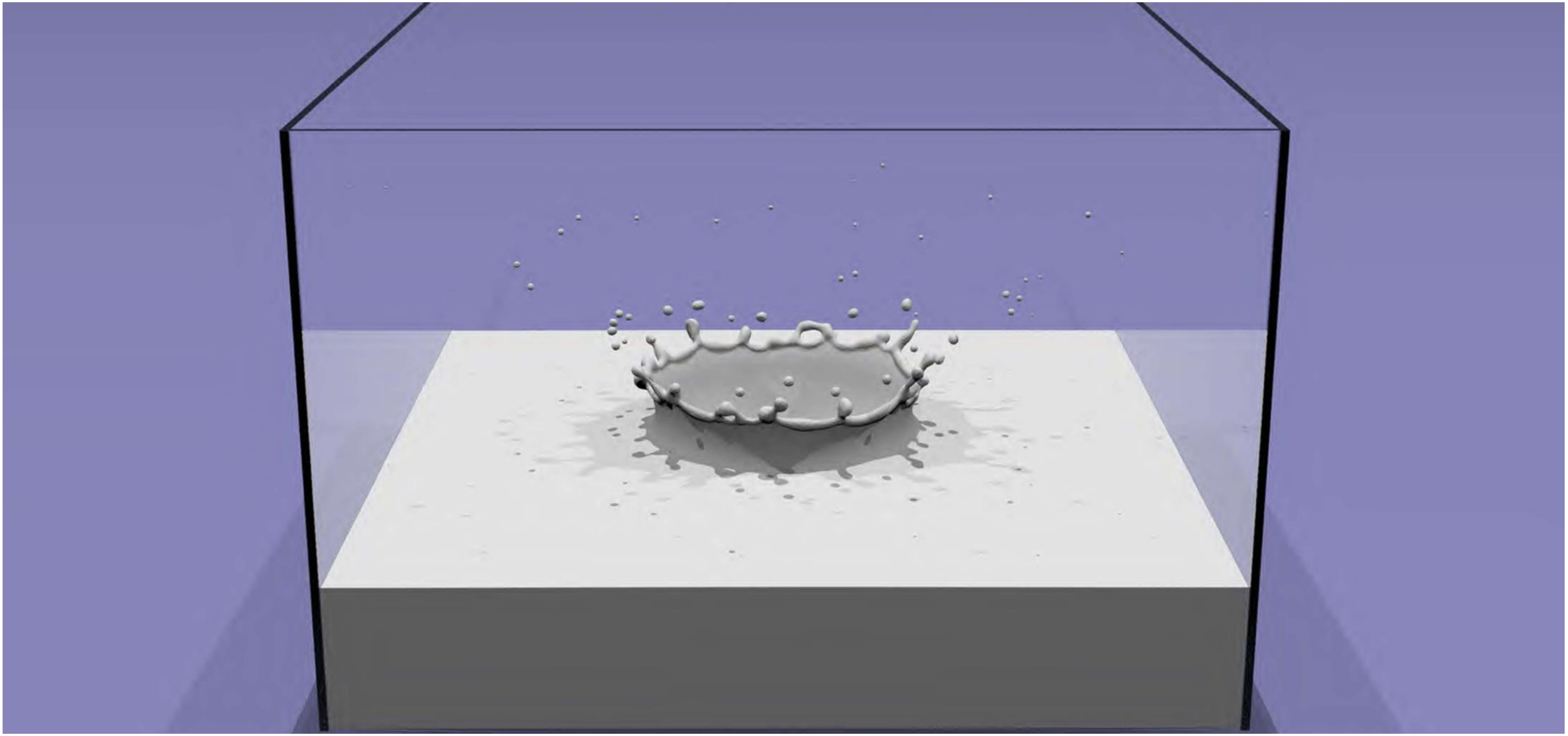
Level Sets from Medical Data (CT, MRI, etc.)

Level sets encode, e.g., constant tissue density



Level Sets in Physical Simulation

Level set encodes distance to air-liquid boundary



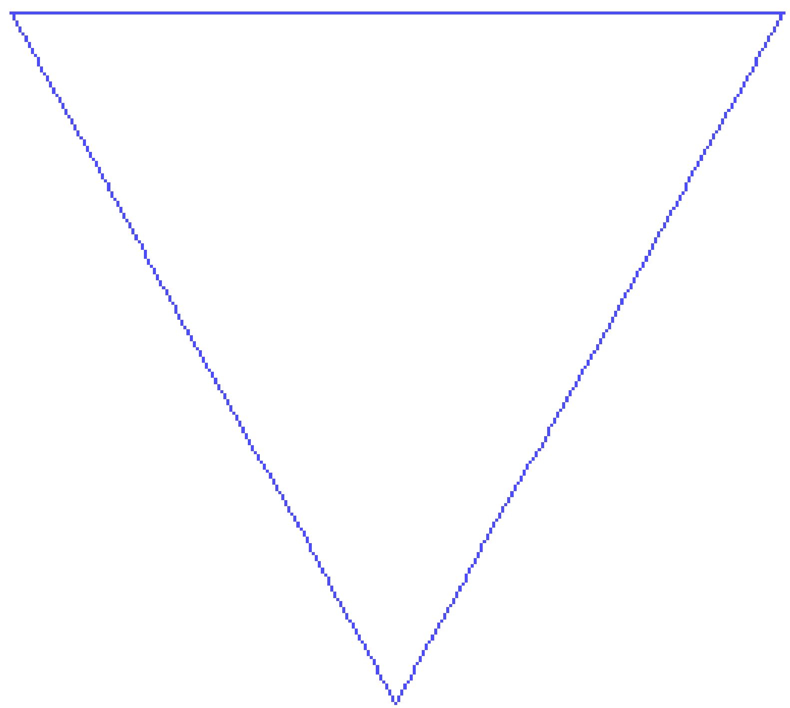
See http://physbam.stanford.edu

Fractals (Implicit)

Exhibit self-similarity, detail at all scales

“Language” for describing natural phenomena

Hard to control shape!



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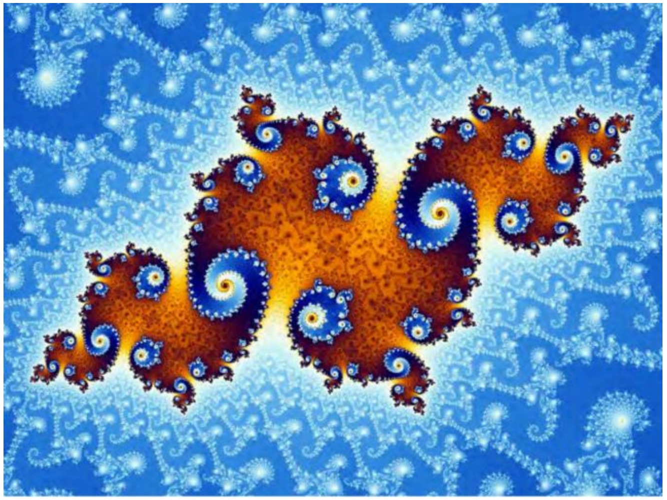

Implicit Representations - Pros & Cons

Pros:

- compact description (e.g., a function)
- certain queries easy (inside object, distance to surface)
- good for ray-to-surface intersection (more later)
- for simple shapes, exact description / no sampling error
- easy to handle changes in topology (e.g., fluid)

Cons:

- difficult to model complex shapes

Thank you!