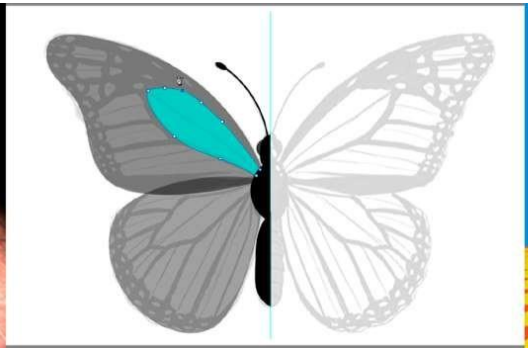
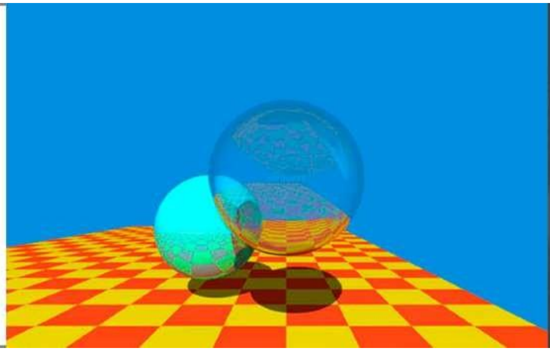
Computer Graphics

Ray Tracing 2
(Radiometry & Light Transport & Global Illumination)


Last Lecture

- Why ray tracing?
- Whitted-style ray tracing
- Ray-object intersections
  - Implicit surfaces
  - Triangles
- Axis-Aligned Bounding Boxes (AABBs)
  - Understanding — pairs of slabs
  - Ray-AABB intersection
- Uniform Spatial Partitions (Grids)
- Oct-Tree KD-Tree BSP-Tree BVH

Why Ray Tracing?

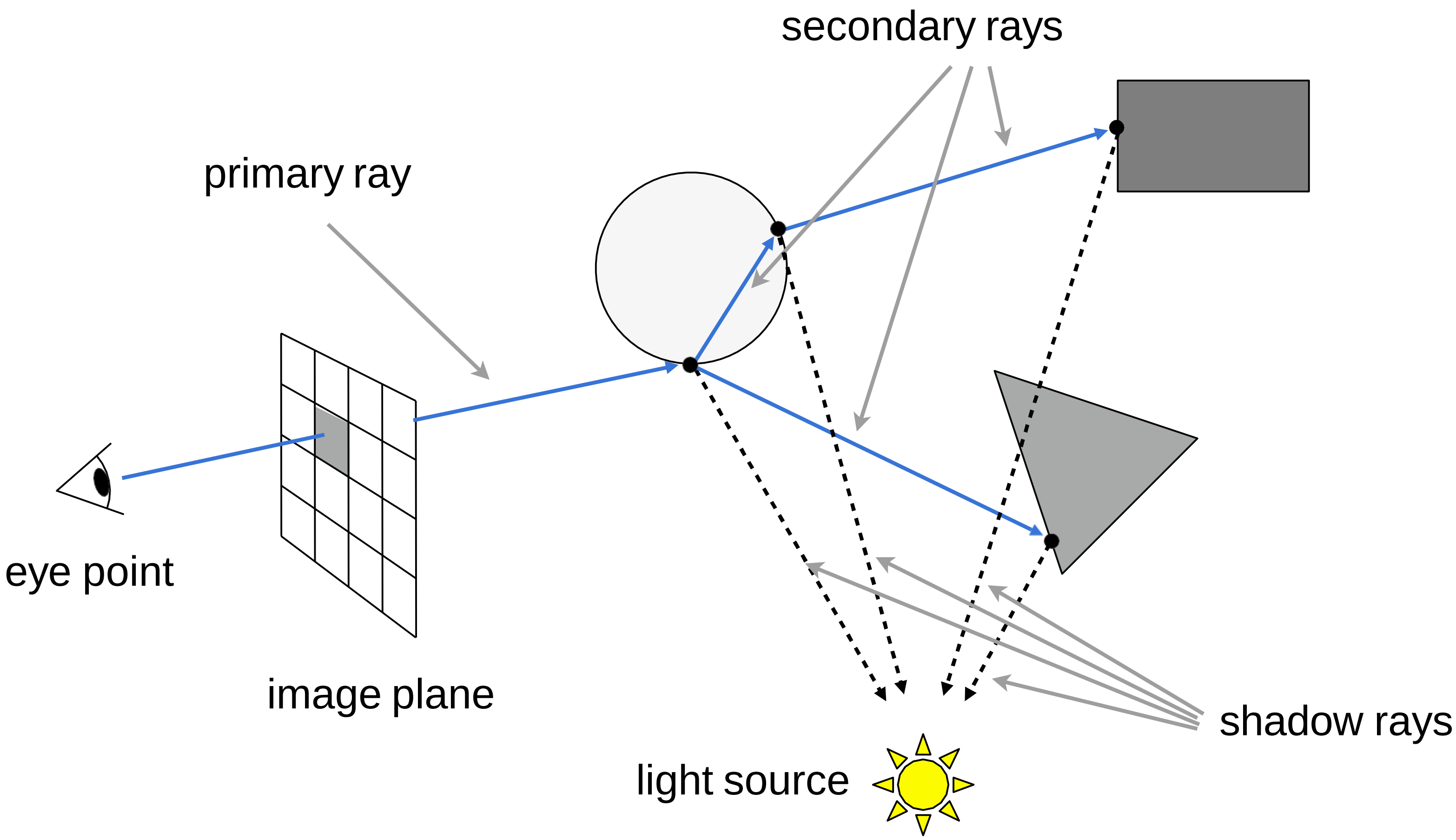
- Rasterization couldn't handle global effects well
  - (Soft) shadows
  - And especially when the light bounces more than once

Soft shadows

Glossy reflection

Indirect illumination

Recursive Ray Tracing

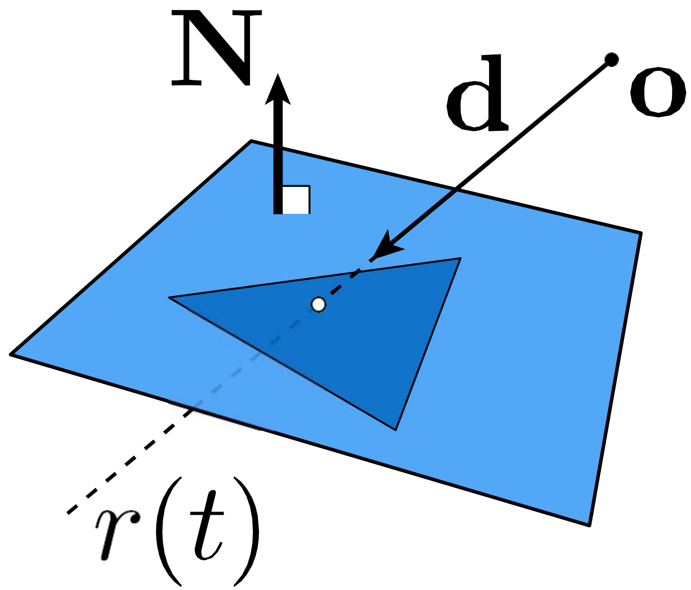


Ray Intersection With Triangle

Triangle is in a plane

- Ray-plane intersection
- Test if hit point is inside triangle

Many ways to optimize…



Möller Trumbore Algorithm

A faster approach, giving barycentric coordinate directly

Derivation in the discussion section!

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2$$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \bullet \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \bullet \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \bullet \vec{\mathbf{S}} \\ \vec{\mathbf{S}}_2 \bullet \vec{\mathbf{D}} \end{bmatrix}$$

Cost = (1 div, 27 mul, 17 add)

Where:

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{E}}_2 = \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{S}} = \vec{\mathbf{O}} - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{S}}_1 = \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2$$
$$\vec{\mathbf{S}}_2 = \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1$$

Recall: How to determine if the “intersection” is inside the triangle?

Hint:
(1-b1-b2), b1, b2 are barycentric coordinates!

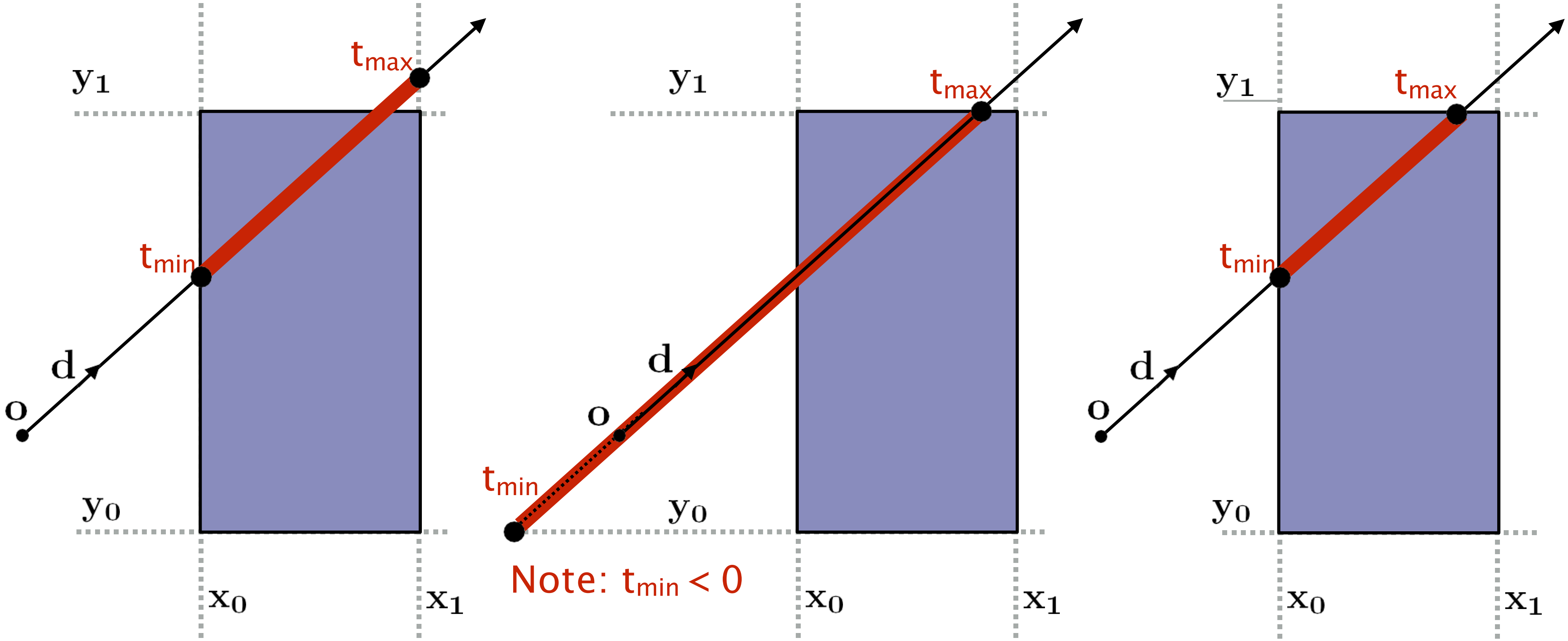
Bounding Volumes

Quick way to avoid intersections: bound complex object with a simple volume

- Object is fully contained in the volume
- If it doesn't hit the volume, it doesn't hit the object
- So test BVol first, then test object if it hits

Ray Intersection with Axis-Aligned Box

2D example; 3D is the same! Compute intersections with slabs and take intersection of t_{min}/t_{max} intervals



Intersections with x planes — Intersections with y planes — Final intersection result

How do we know when the ray intersects the box?

Ray Intersection with Axis-Aligned Box

- Recall: a box (3D) = three pairs of infinitely large slabs
- Key ideas
  - The ray enters the box only when it enters all pairs of slabs
  - The ray exits the box as long as it exits any pair of slabs
- For each pair, calculate the t_{min} and t_{max} (negative is fine)
- For the 3D box, $t_{enter} = max\{t_{min}\}$, $t_{exit} = min\{t_{max}\}$
- If $t_{enter} < t_{exit}$, we know the ray stays awhile in the box (so they must intersect!) (not done yet, see the next slide)

Ray Intersection with Axis-Aligned Box

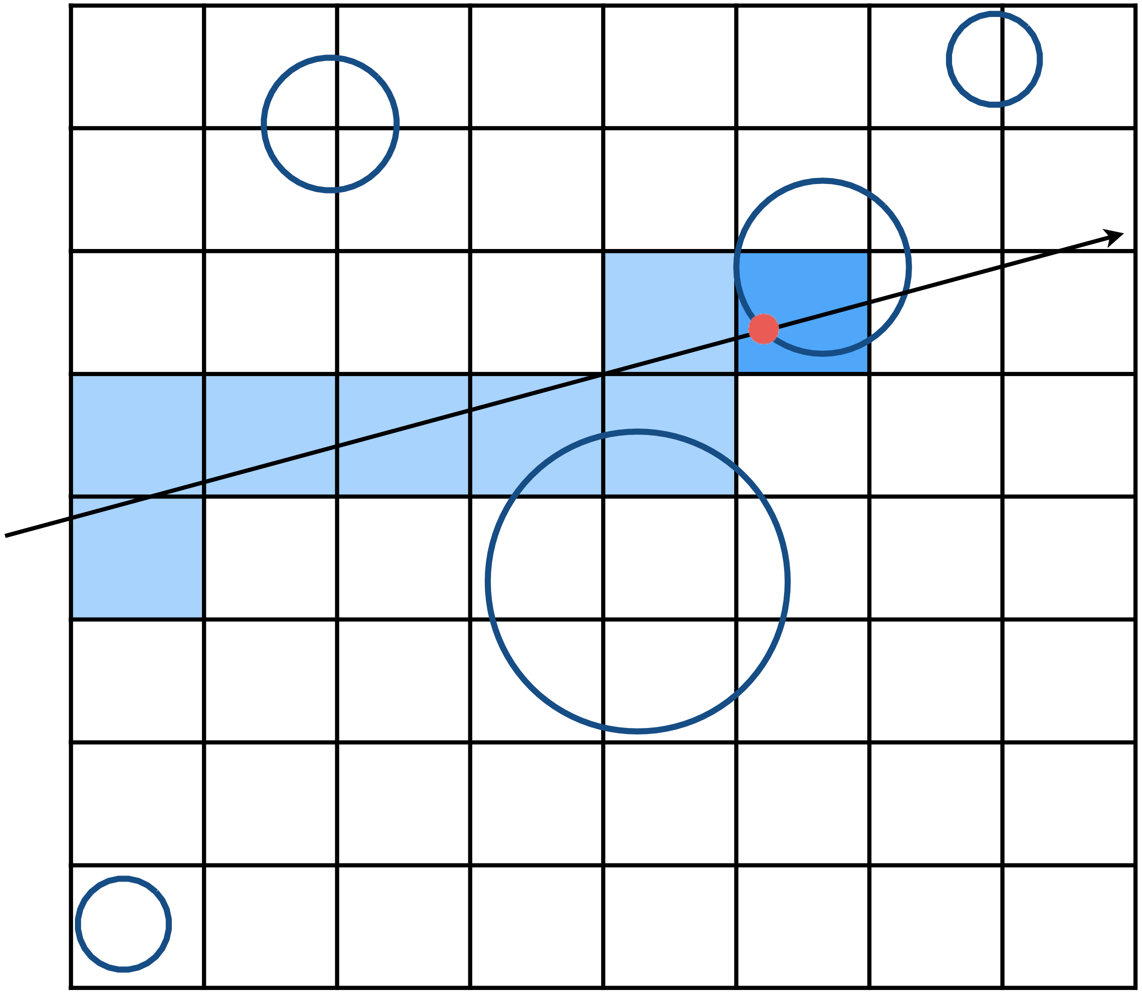
- However, ray is not a line
  - Should check whether t is negative for physical correctness!

- What if $t_{exit} < 0$?
  - The box is "behind" the ray — no intersection!

- What if $t_{exit} >= 0$ and $t_{enter} < 0$?
  - The ray's origin is inside the box — have intersection!

- In summary, ray and AABB intersect iff
  - $t_{enter} < t_{exit}$ && $t_{exit} >= 0$

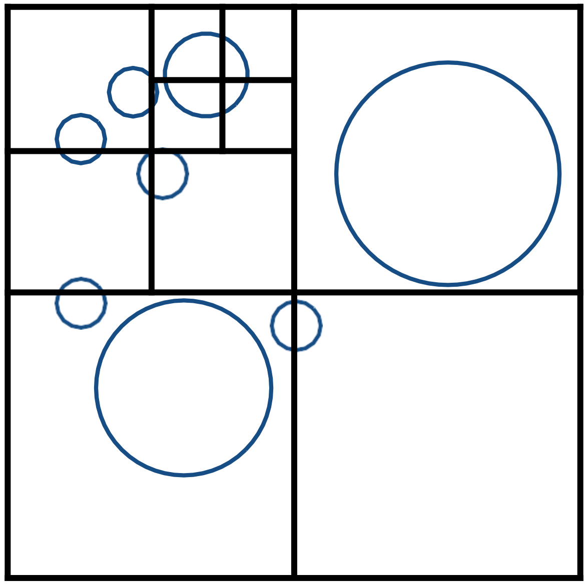
Ray-Scene Intersection



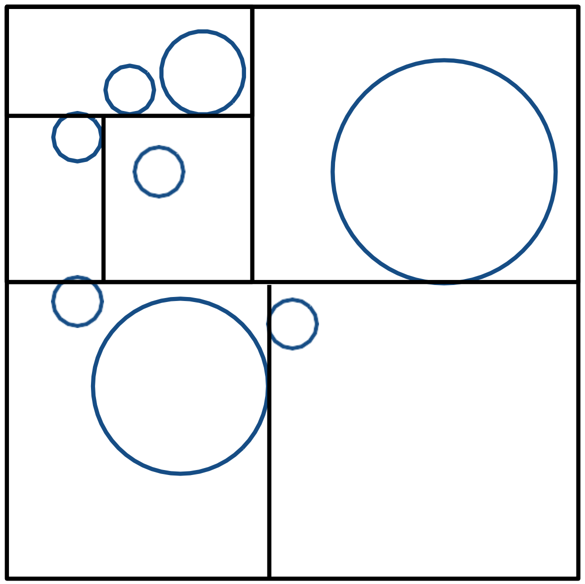
Step through grid in ray traversal order

For each grid cell
    Test intersection with all objects stored at that cell

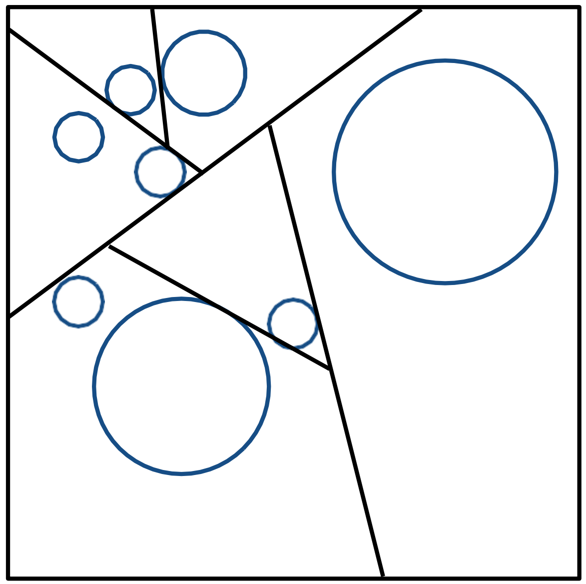
Spatial Partitioning Examples



Oct-Tree



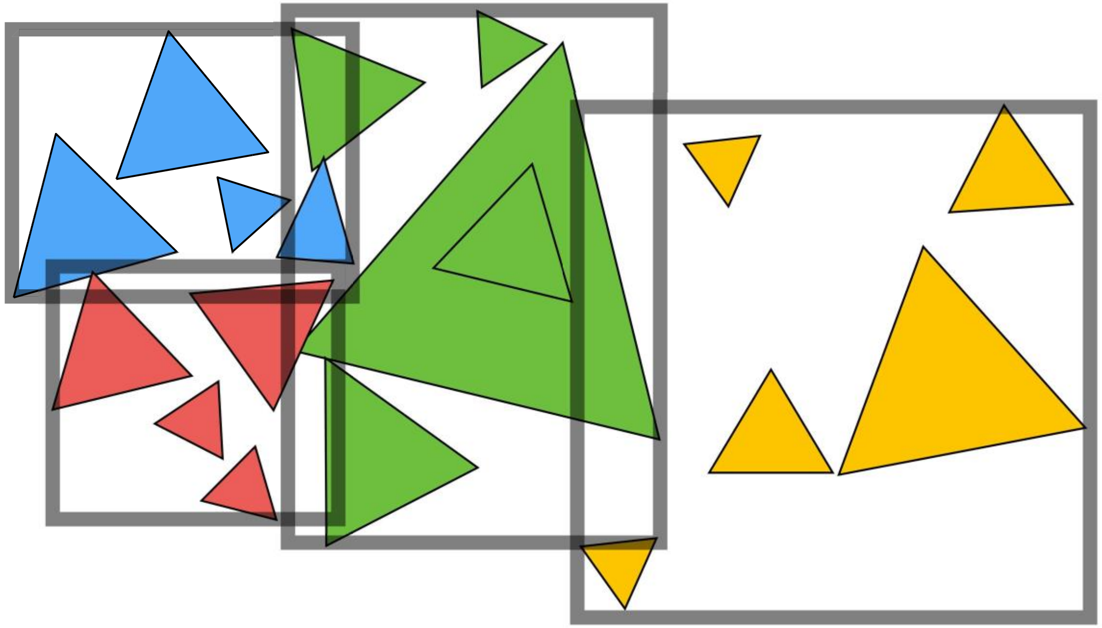
KD-Tree



BSP-Tree

Note: you could have these in both 2D and 3D. In lecture we will illustrate principles in 2D.

Summary: Building BVHs



- Find bounding box
- Recursively split set of objects in two subsets
- Recompute the bounding box of the subsets
- Stop when necessary
- Store objects in each leaf node

Data Structure for BVHs

Internal nodes store

- Bounding box
- Children: pointers to child nodes

Leaf nodes store

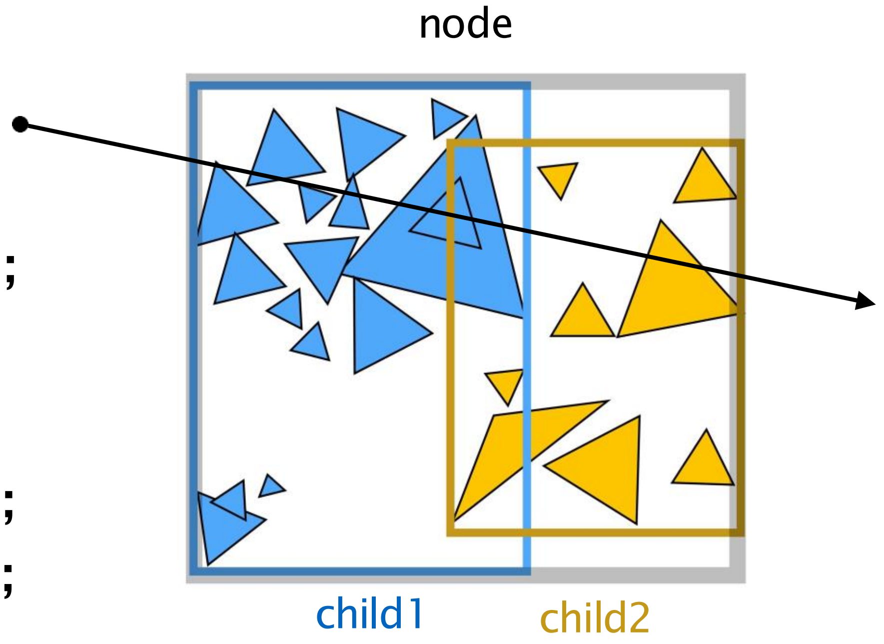
- Bounding box
- List of objects

Nodes represent subset of primitives in scene

- All objects in subtree

BVH Traversal

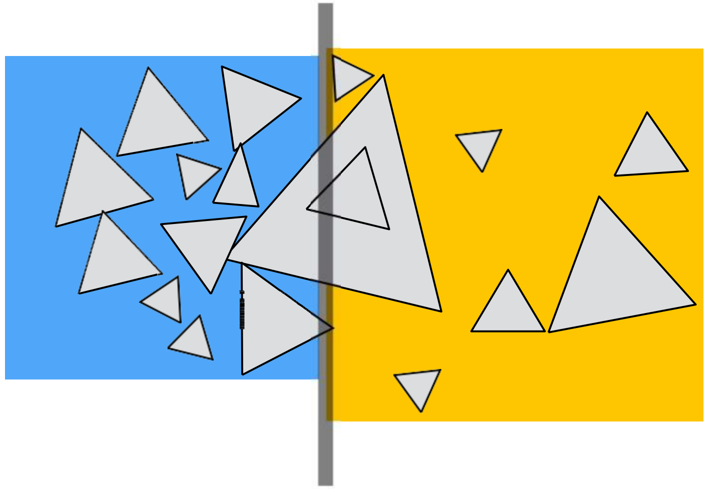
```
Intersect(Ray ray, BVH node) {
  if (ray misses node.bbox) return;

  if (node is a leaf node)
     test intersection with all objs;
     return closest intersection;

  hit1 = Intersect(ray, node.child1);
  hit2 = Intersect(ray, node.child2);

  return the closer of hit1, hit2;
}
```



Spatial vs Object Partitions

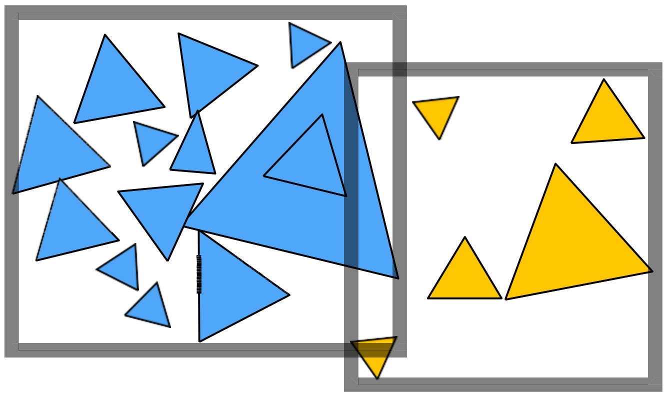
Spatial partition (e.g.KD-tree)

- Partition space into non-overlapping regions
- An object can be contained in multiple regions
- Intersection between objects and bounding box



Object partition (e.g. BVH)

- Partition set of objects into disjoint subsets
- Bounding boxes for each set may overlap in space



多选题 1分

设置

光线追踪对于光线的物理性质有哪些基本假设?

A 光沿直线传播

B 忽略光的波动属性

C 光线不会发生碰撞

D 光线的可逆性

E 光线发生碰撞能量会有损失

提交

请问AABB包围盒与OBB包围盒的优缺点分别是?

作答

Today

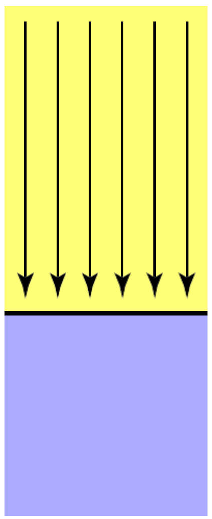
- Using AABBs to accelerate ray tracing
  - Uniform grids
  - Spatial partitions
- Basic radiometry （辐射度量学）
  - Advertisement: new topics from now on, scarcely covered in other graphics courses


CREE
soft white
LED
60w
replacement
uses only 11 Watts
4FLOW™
— FILAMENT DESIGN —
LOOKS AND LIGHTS LIKE A LIGHT BULB
Brightness
815
lumens
Estimated
Energy Cost
$1.32
per year
Dimmable
CREE

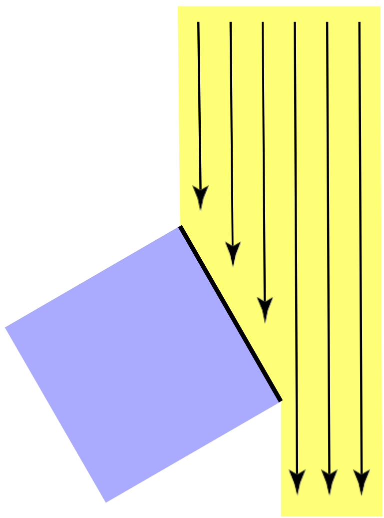
四季，能量

Diffuse Reflection

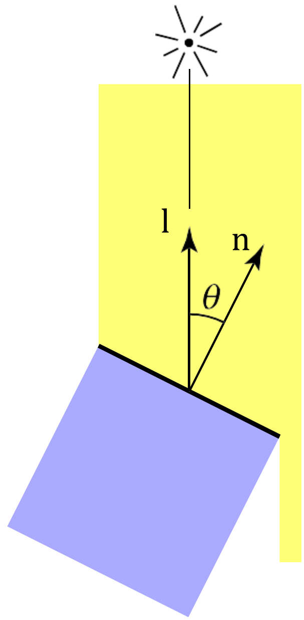
- But how much light (energy) is received?
  - Lambert's cosine law



Top face of cube receives a certain amount of light

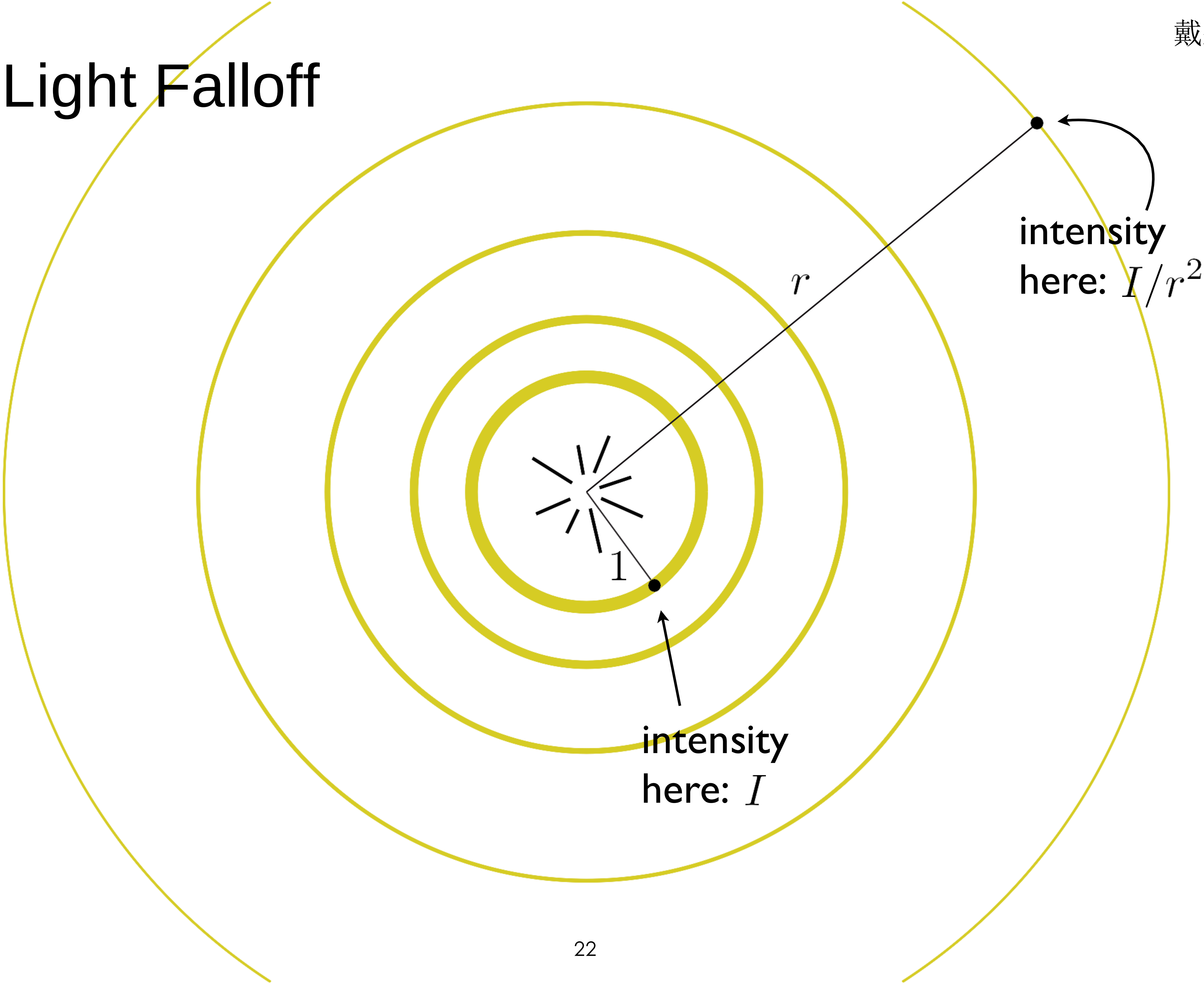


Top face of 60° rotated cube intercepts half the light



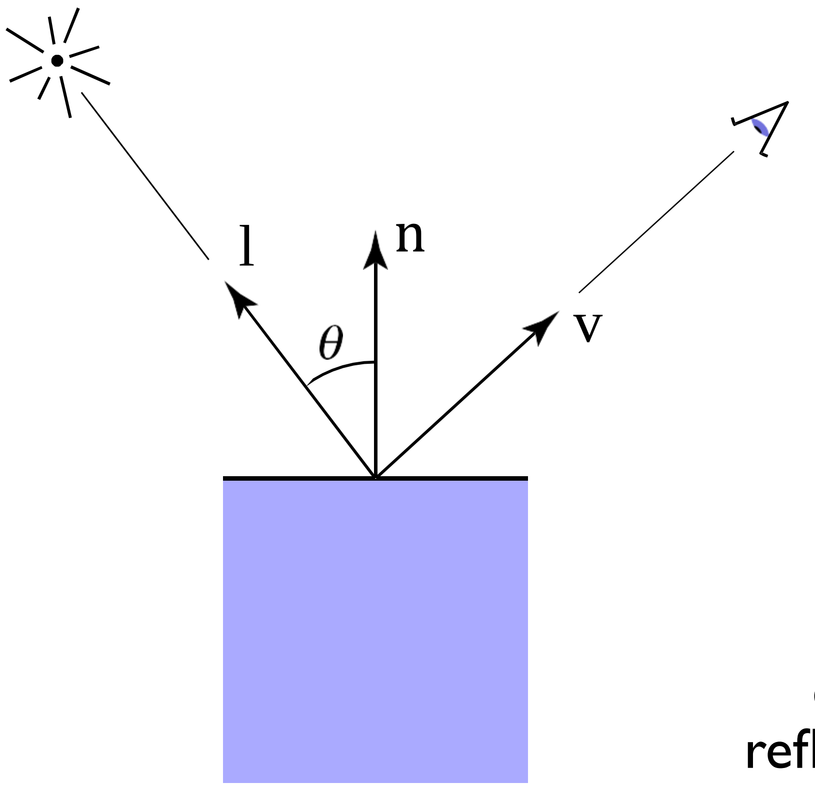
In general, light per unit area is proportional to $\cos\theta = l \cdot n$

Light Falloff



Lambertian (Diffuse) Shading

Shading independent of view direction



energy arrived at the shading point

$$L_d = k_d \, (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{l})$$

diffusely reflected light

diffuse coefficient (color)

energy received by the shading point

Radiometry — Motivation

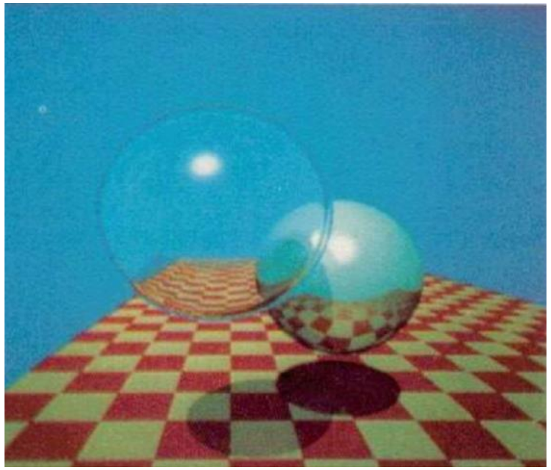
Observation

- In assignment 3, we implement the Blinn-Phong model
- Light intensity I is 10, for example
- But 10 what?

Do you think Whitted style ray tracing gives you CORRECT results?

All the answers can be found in radiometry

- Also the basics of “Path Tracing”



Radiometry (辐射度量学)

Measurement system and units for illumination

Accurately measure the spatial properties of light

- New terms: Radiant flux, intensity, irradiance, radiance

Perform lighting calculations in a physically correct manner

My personal way of learning things:

- WHY, WHAT, then HOW

Radiant Energy and Flux (Power)

辐射能　　　　辐射通量

Radiant Energy and Flux (Power)

辐射能　　　　　　　　辐射通量

Definition: Radiant energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

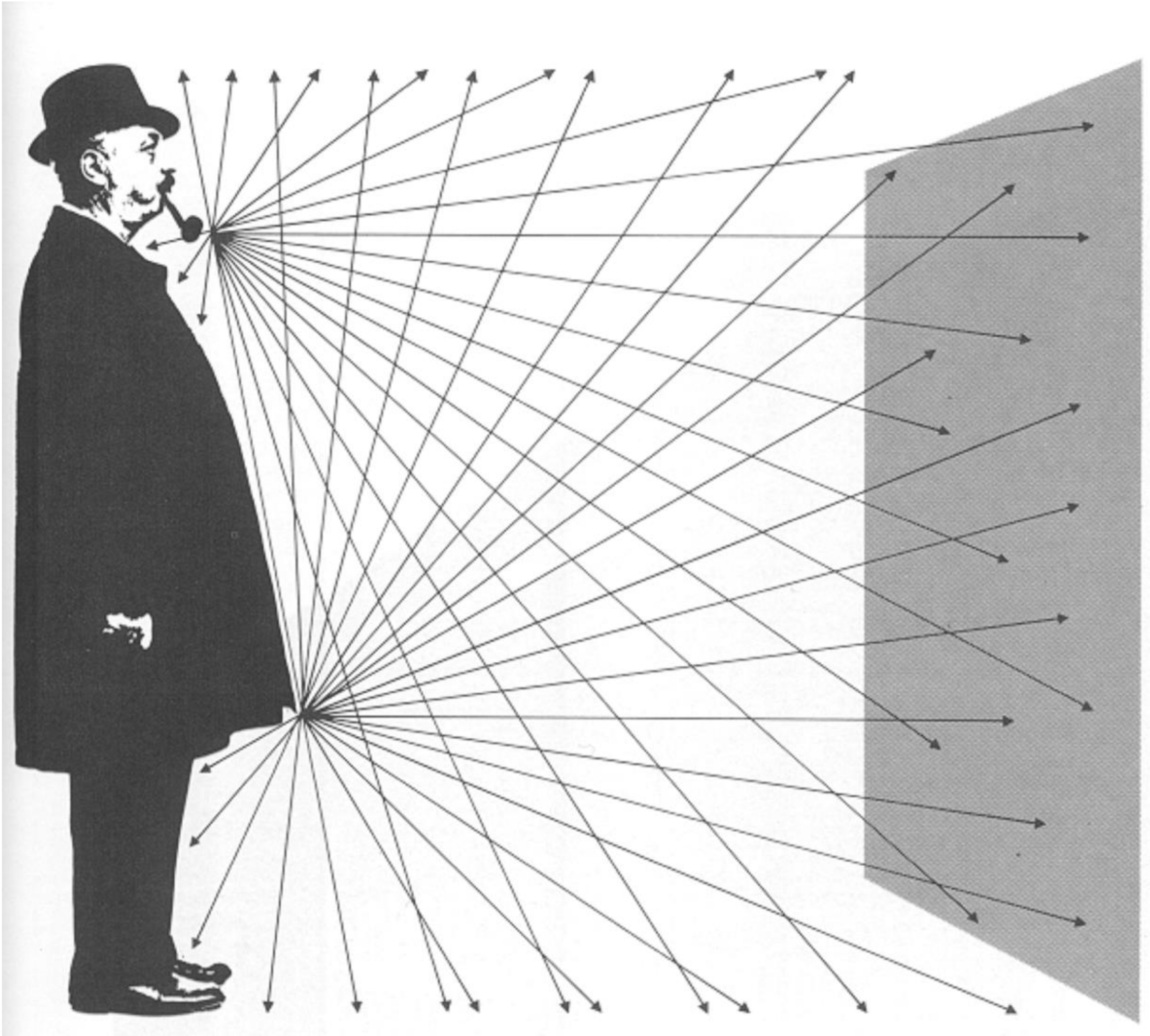
$$Q \ [\mathrm{J} = \mathrm{Joule}]$$

Definition: Radiant flux (power) is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \ [\mathbf{W} = \mathrm{Watt}] \ [\mathrm{lm} = \mathrm{lumen}]^*$$

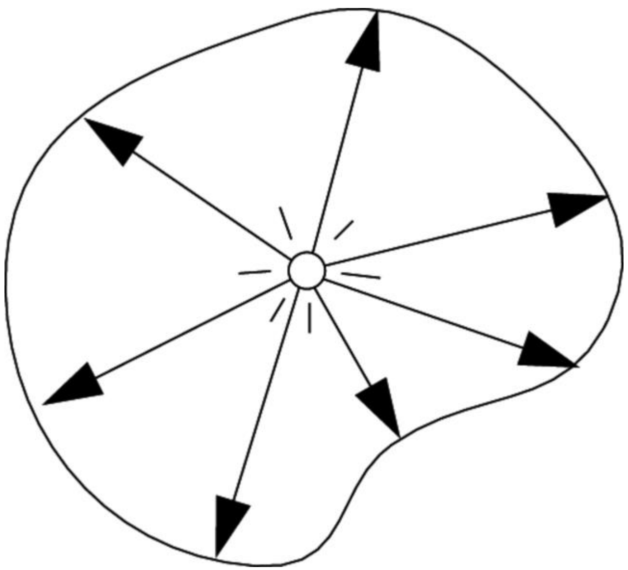
流明

Flux – #photons flowing through a sensor in unit time



From London and Upton

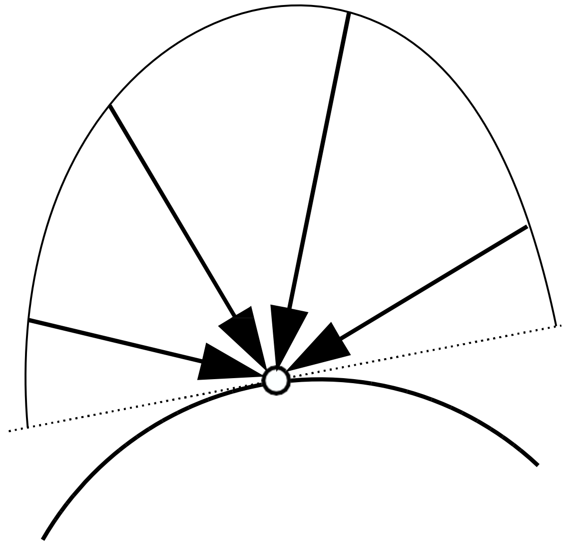
Important Light Measurements of Interest



Light Emitted
From A Source

"Radiant Intensity"

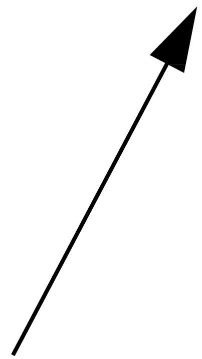
辐射强度



Light Falling
On A Surface

"Irradiance"

辐照度



Light Traveling
Along A Ray

"Radiance"

辐射

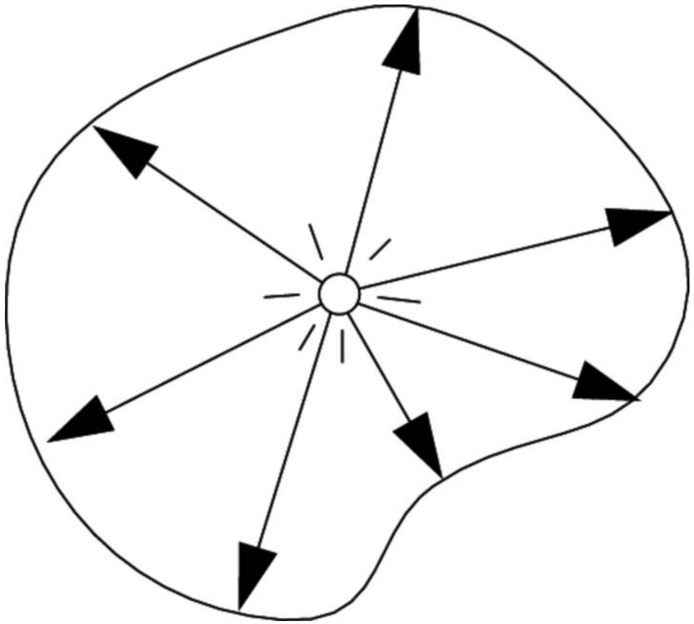
Radiant Intensity

辐射强度

Radiant Intensity 辐射强度

Definition: The radiant (luminous) intensity is the power per **unit solid angle** (?) emitted by a point light source.

(立体角)



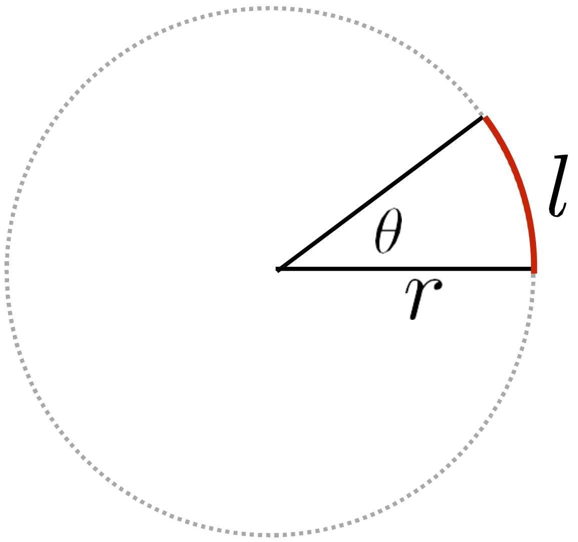
$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

烛光或坎德拉（英语拉丁语：candela）是发光强度的单位，国际单位制七大基本单位之一，符号cd

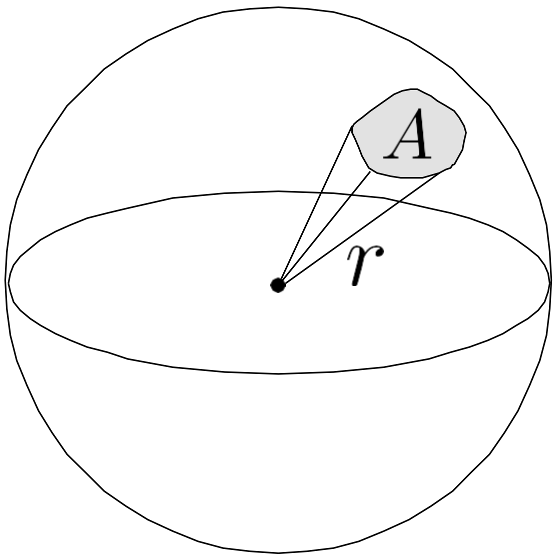
Angles and Solid Angles

Angle: ratio of subtended arc length on circle to radius



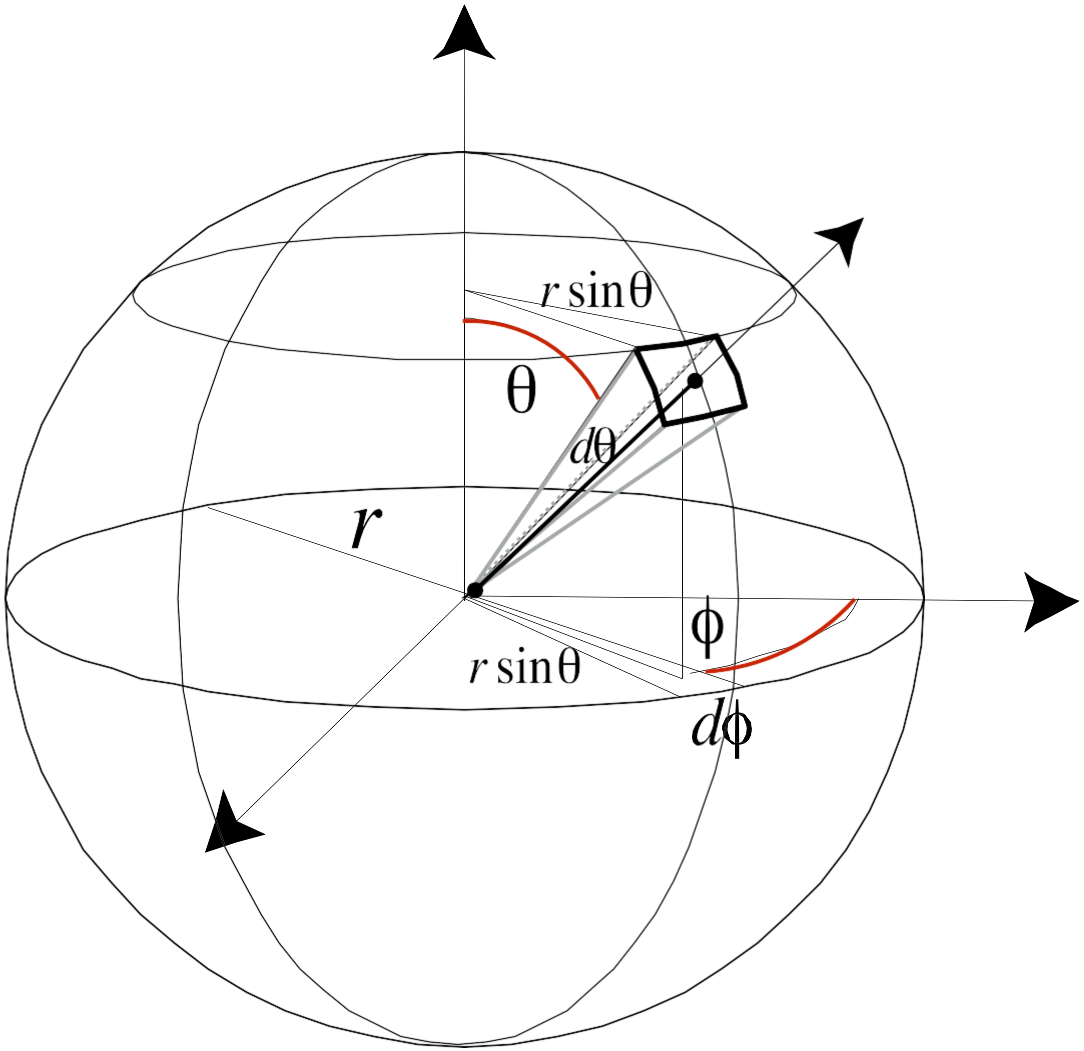
- $\theta = \frac{l}{r}$
- Circle has 2π radians

Solid angle: ratio of subtended area on sphere to radius squared



- $\Omega = \frac{A}{r^2}$
- Sphere has 4π steradians

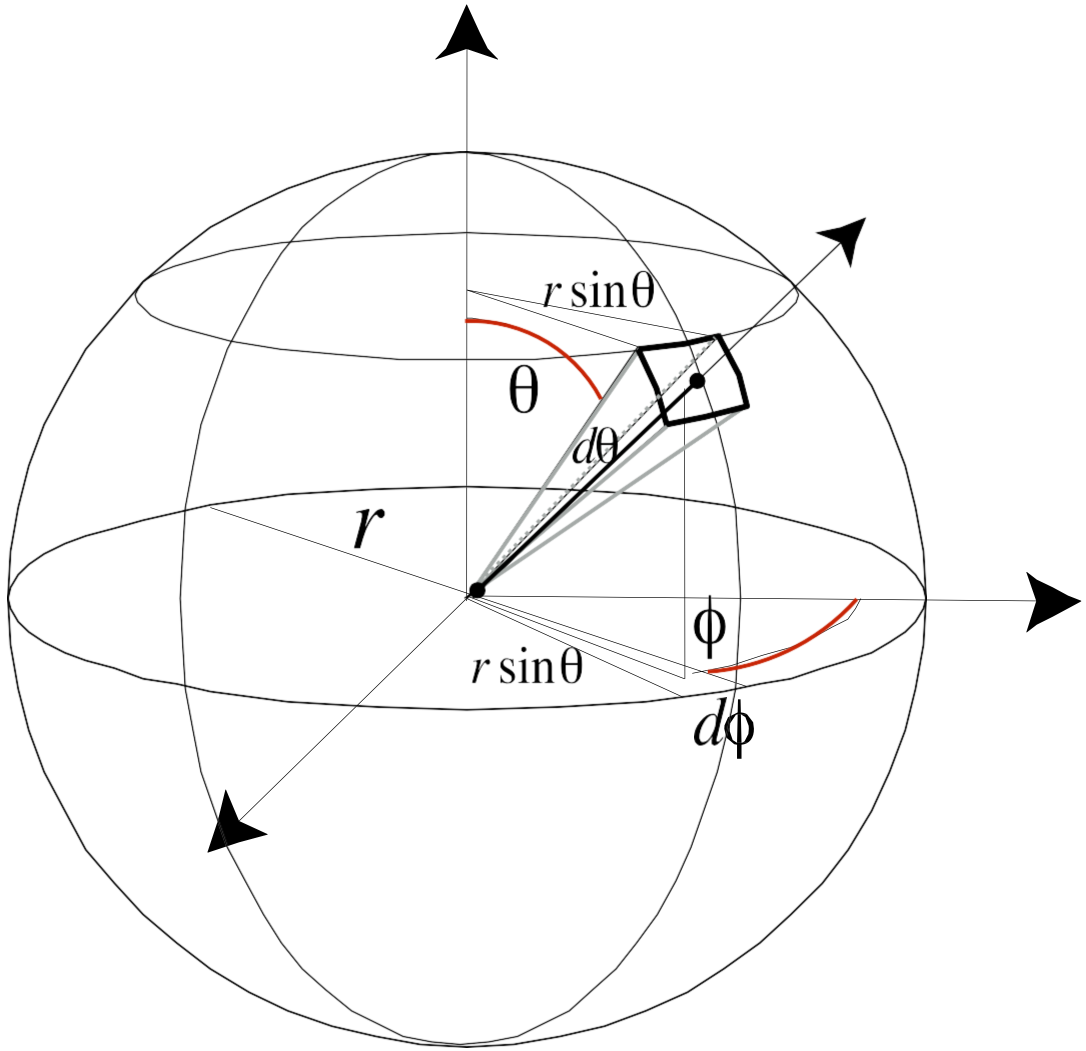
Differential Solid Angles



$$\begin{aligned} \mathrm{d}A =& (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi) \\ =& r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \end{aligned}$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Differential Solid Angles



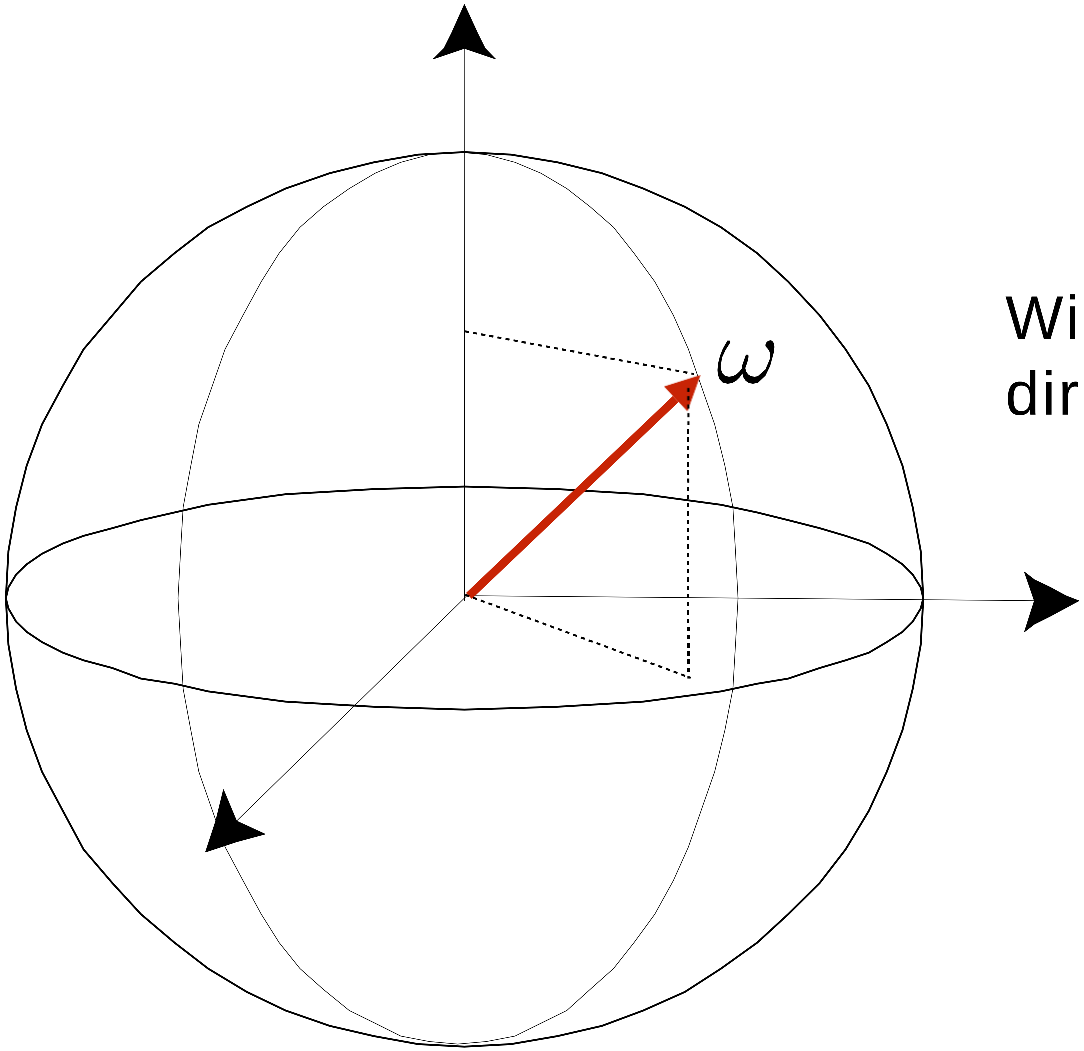
Sphere: S^2

$$\Omega = \int_{S^2} \mathrm{d}\omega$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

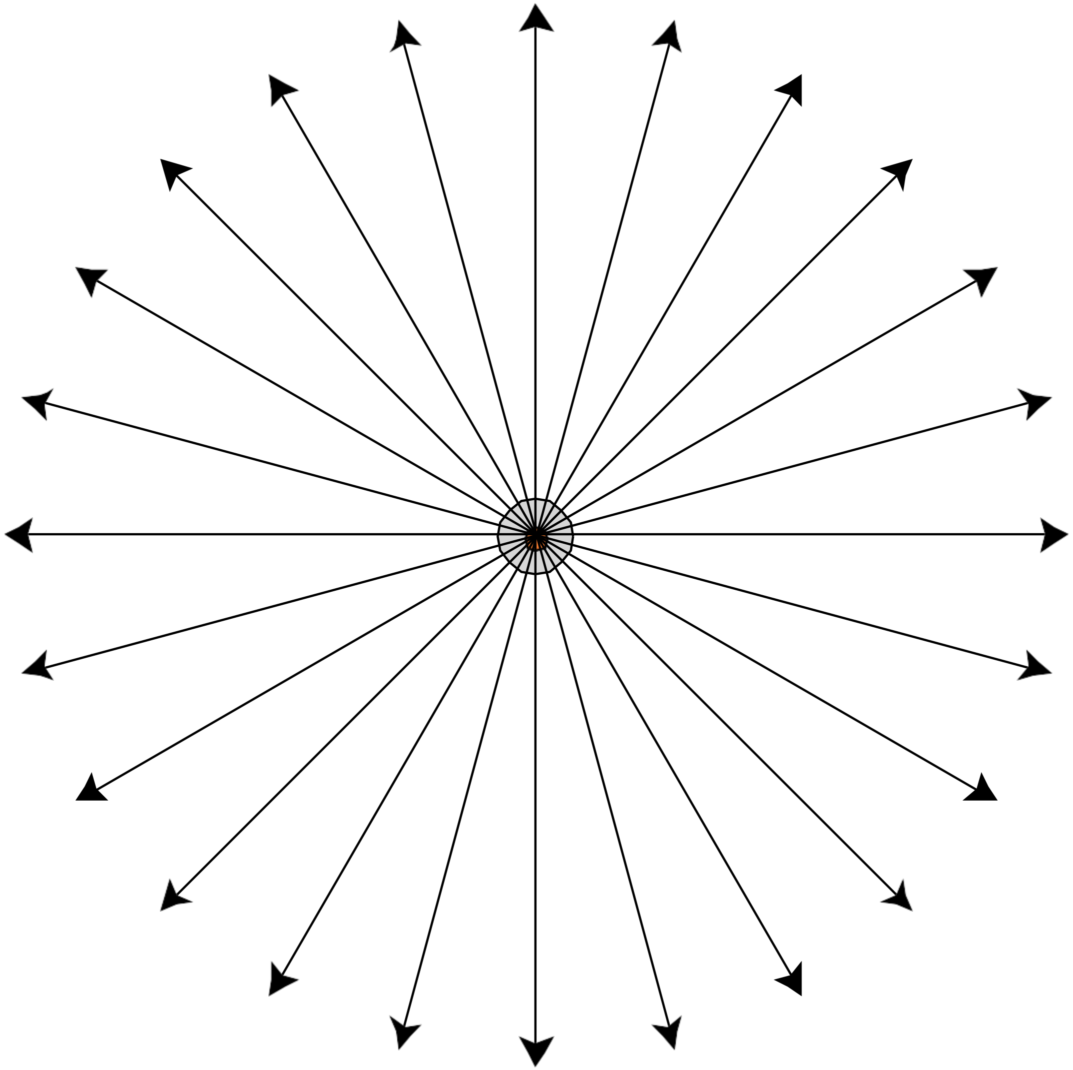
$$= 4\pi$$

ω as a direction vector



Will use ω to denote a direction vector (unit length)

Isotropic Point Source



$$\Phi = \int_{S^2} I \, \mathrm{d}\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Modern LED Light

Output: 815 lumens

(11W LED replacement
for 60W incandescent)

Radiant intensity?

Assume isotropic:

Intensity = 815 lumens / 4pi sr
= 65 candelas

Reviewing Concepts

辐射能

Radiant energy $Q \; [\mathrm{J} = \mathrm{Joule}]$ (barely used in CG)

- the energy of electromagnetic radiation

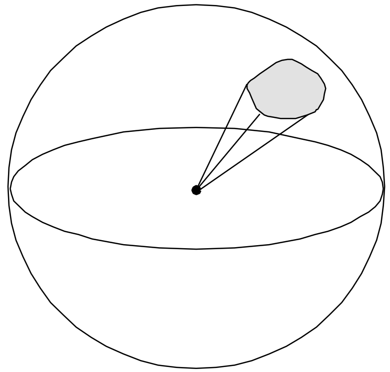
辐射通量

Radiant flux (power) $\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \; [\mathrm{W} = \mathrm{Watt}] \; [\mathrm{lm} = \mathrm{lumen}]$

- Energy per unit time

辐射强度

Radiant intensity $I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$



- power per unit solid angle

立体角

Solid Angle $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared

Irradiance

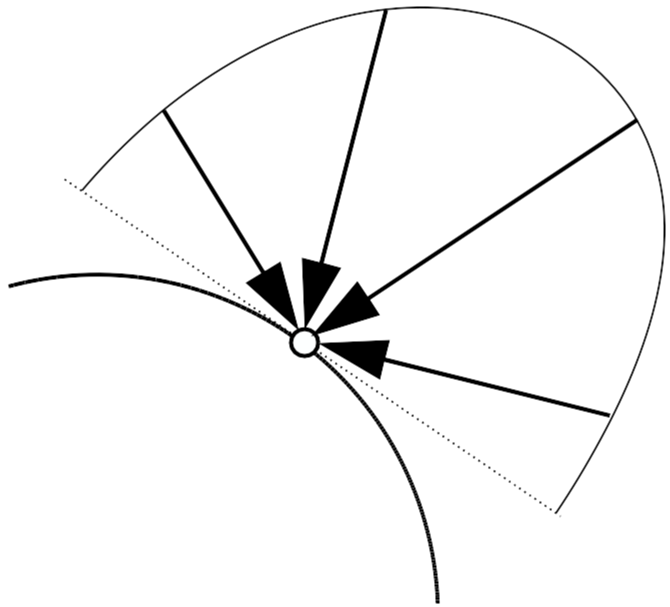
辐照度

Irradiance 辐照度

Definition: The irradiance is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

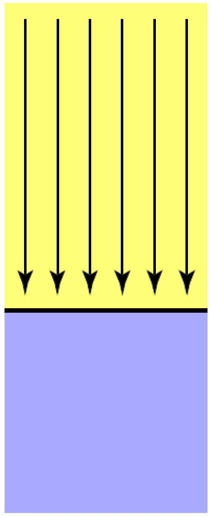
$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \quad \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$



Lambert's Cosine Law

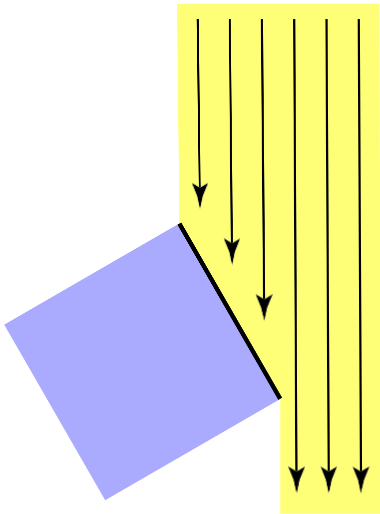
Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

(Note: always use a unit area, the cosine applies on Φ)



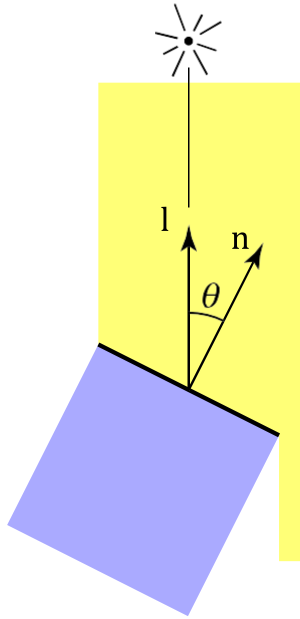
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

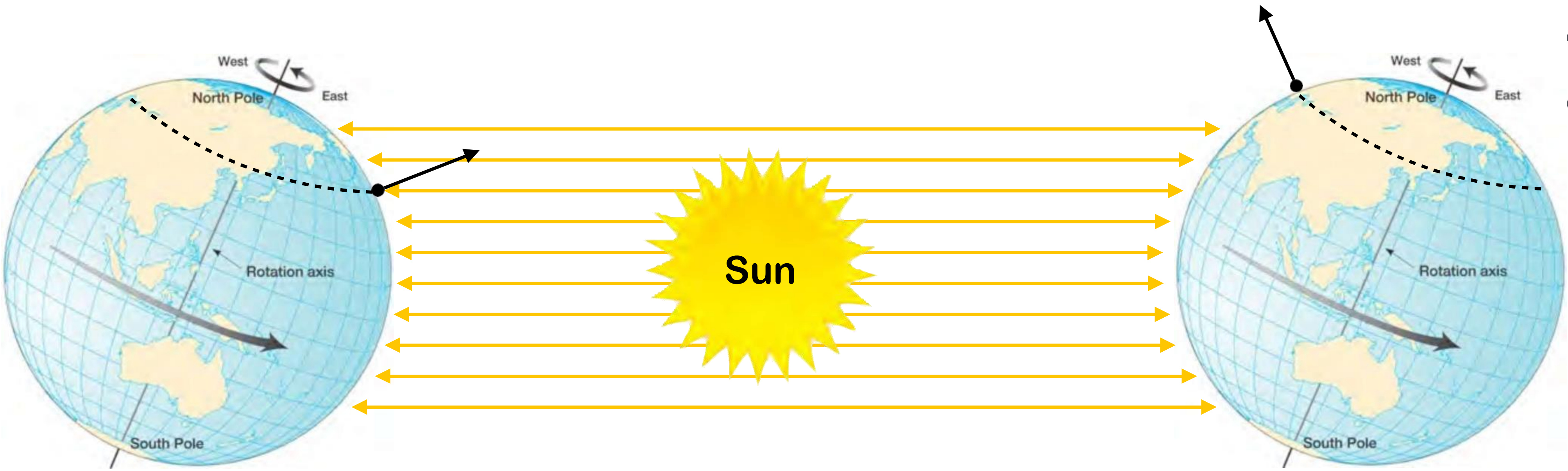
$$E = \frac{1}{2}\frac{\Phi}{A}$$



In general, power per unit area is proportional to $\cos\theta = l \cdot n$

$$E = \frac{\Phi}{A}\cos\theta$$

Why Do We Have Seasons?



[Image credit: Pearson Prentice Hall]

Summer
(Northern hemisphere)

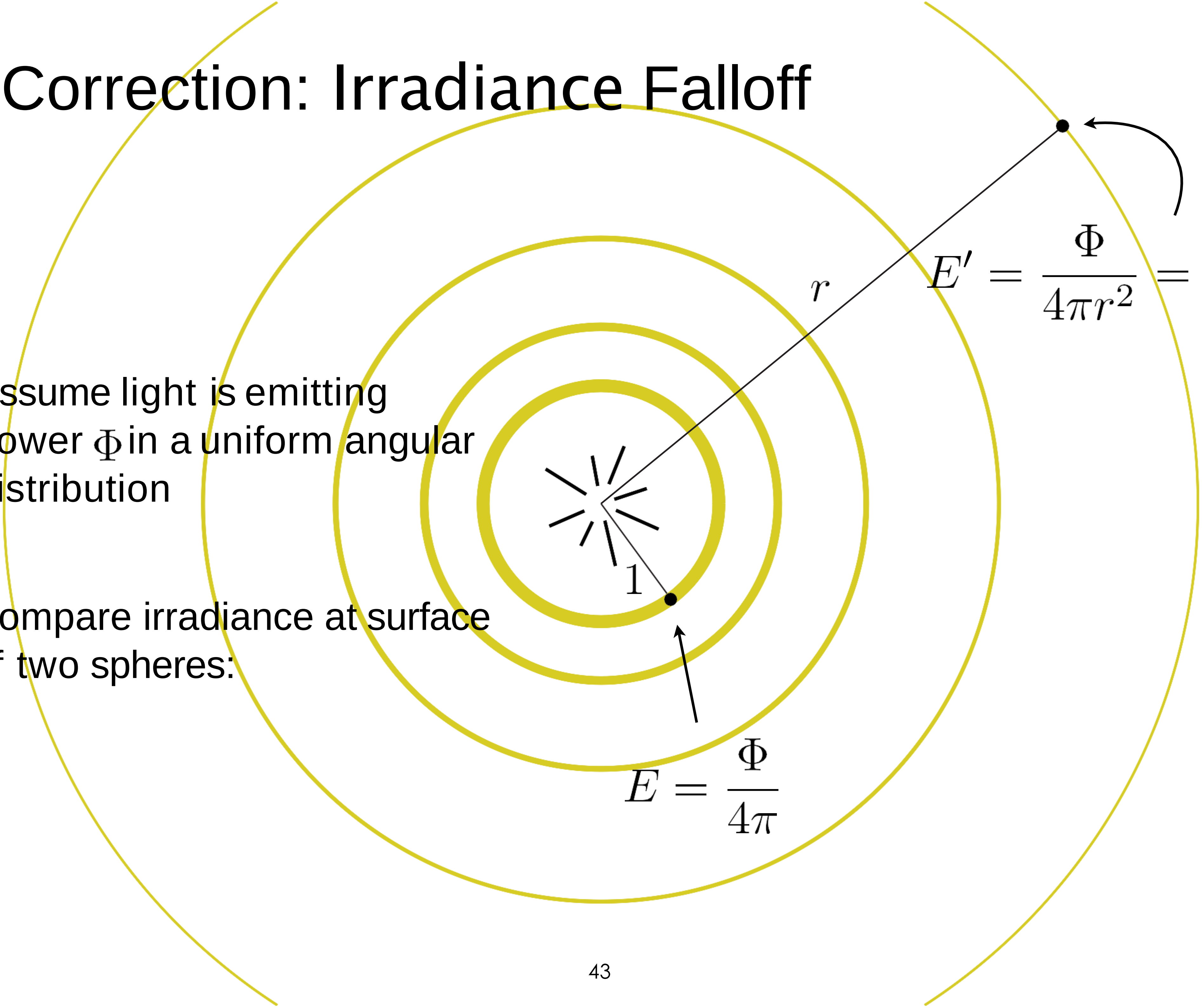
Winter
(Northern hemisphere)

Earth’s axis of rotation: ~23.5° off axis

Correction: Irradiance Falloff

Assume light is emitting power Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:



$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

$$E = \frac{\Phi}{4\pi}$$

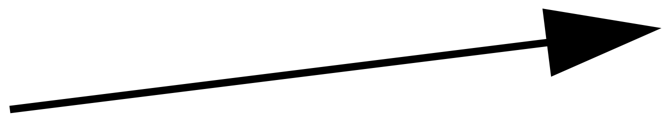
Radiance

辐射

Radiance 辐射

Radiance is the fundamental field quantity that describes the distribution of light in an environment

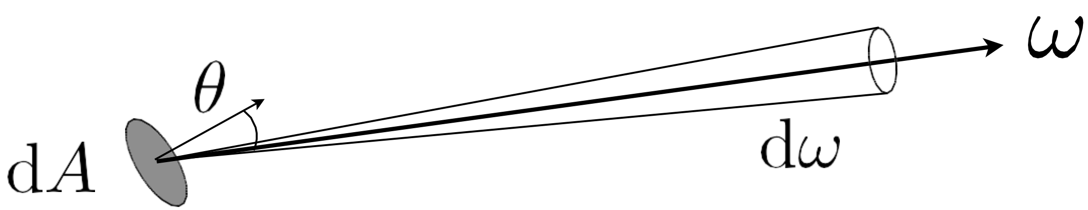
- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance



Light Traveling Along A Ray

Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per projected unit area.



$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr}\,\mathrm{m}^2}\right] \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr}\,\mathrm{m}^2} = \mathrm{nit}\right]$$

Radiance

Definition: power per unit solid angle per projected unit area.

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

Recall

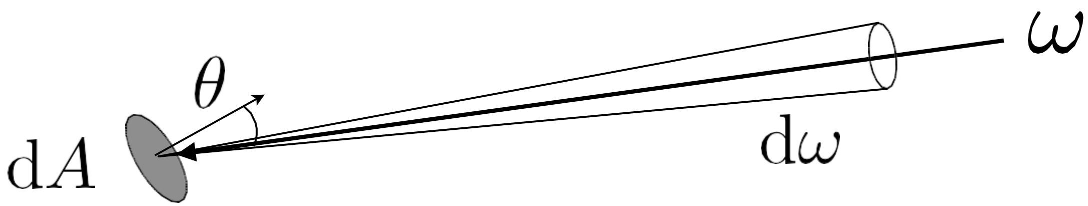
- Irradiance: power per projected unit area
- Intensity: power per solid angle

So

- Radiance: Irradiance per solid angle
- Radiance: Intensity per projected unit area

Incident Radiance

Incident radiance is the irradiance per unit solid angle arriving at the surface.

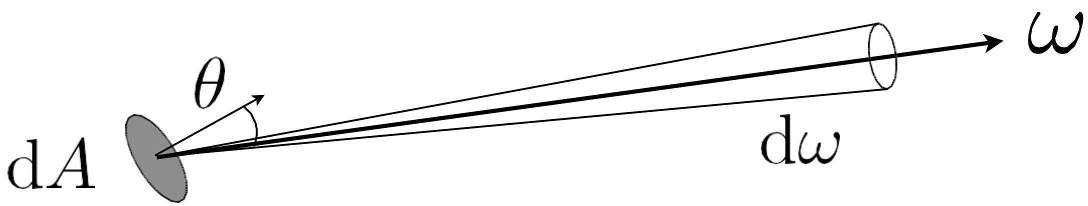


$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Radiance

Exiting surface radiance is the intensity per unit projected area leaving the surface.



$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos\theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Irradiance vs. Radiance

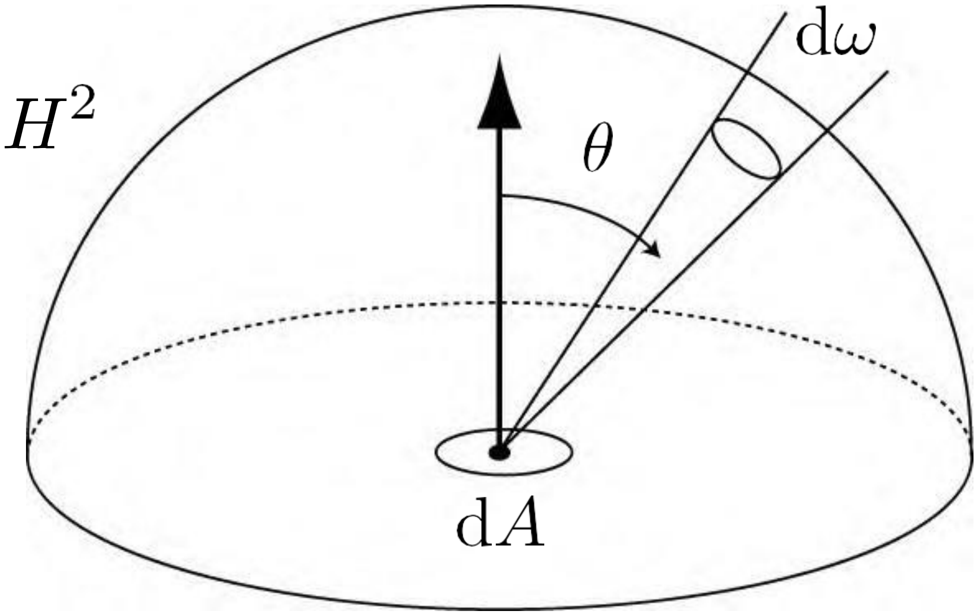
Irradiance: total power received by area dA

Radiance: power received by area dA from “direction” dω

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

Unit Hemisphere: H^2

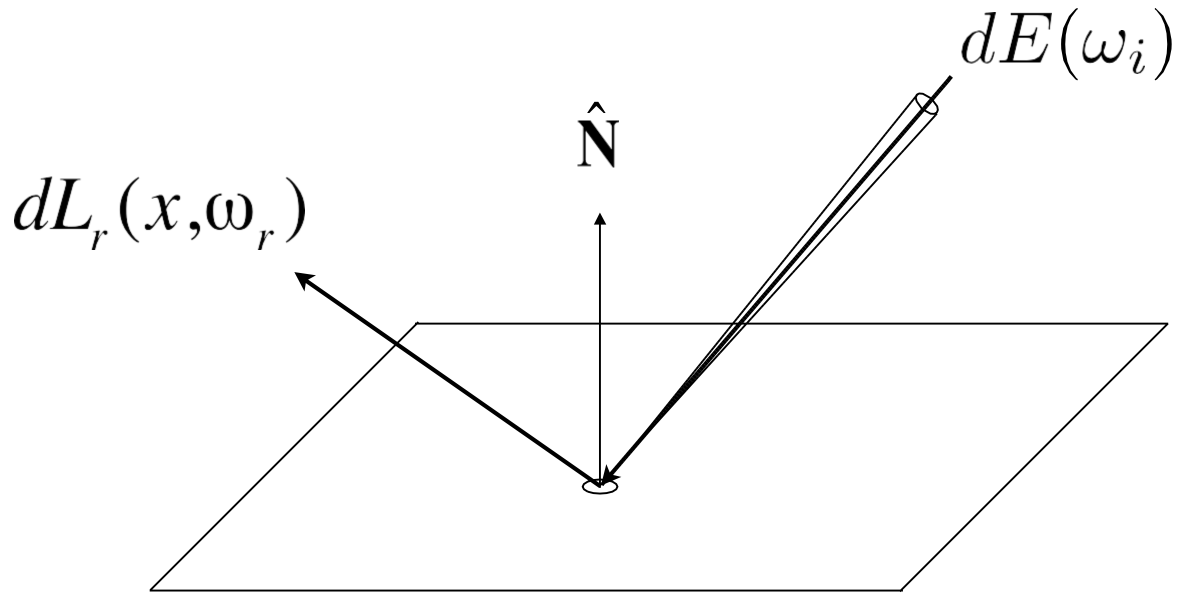


Bidirectional Reflectance Distribution Function (BRDF)

Reflection at a Point

Radiance from direction ω_i turns into the power E that dA receives

Then power E will become the radiance to any other direction ω_o

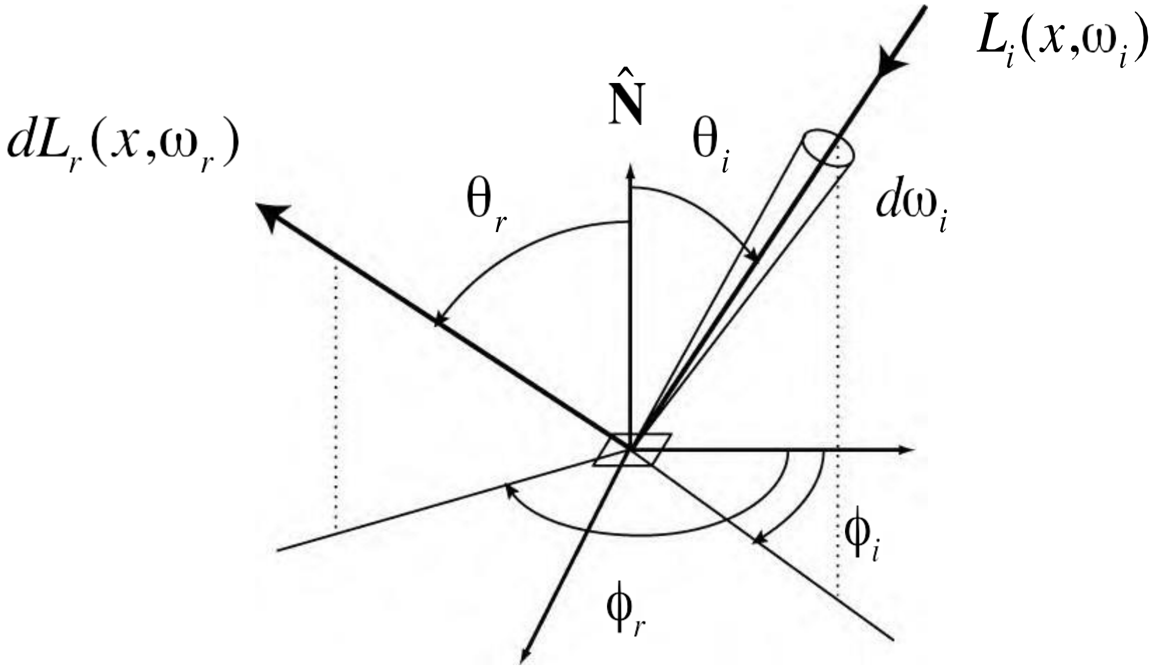


Differential irradiance incoming: $$dE(\omega_i) = L(\omega_i)\, cos\, \theta_i\, d\omega_i$$

Differential radiance exiting (due to $dE(\omega_i)$): $$dL_r(\omega_r)$$

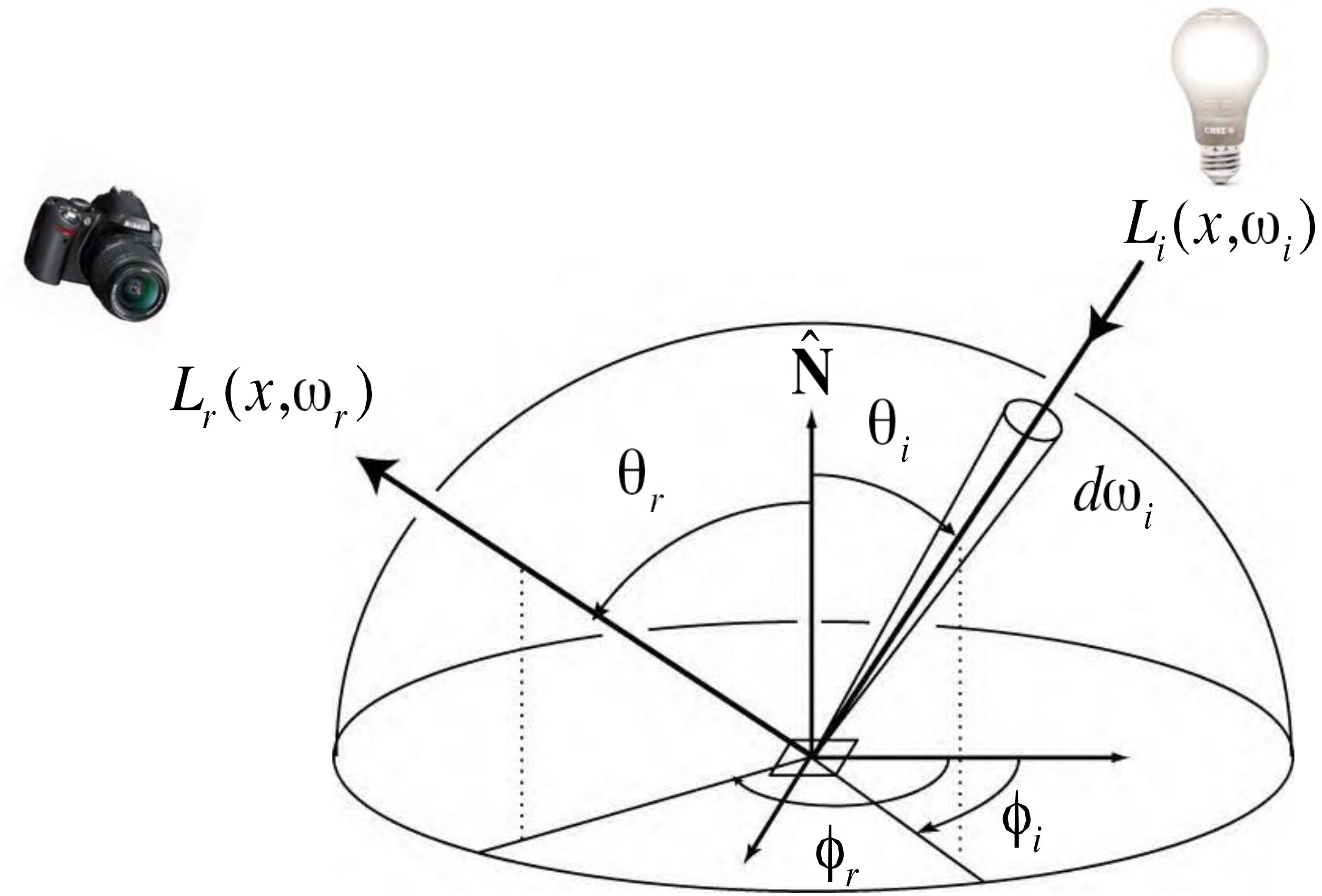
BRDF

The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction



$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

The Reflection Equation

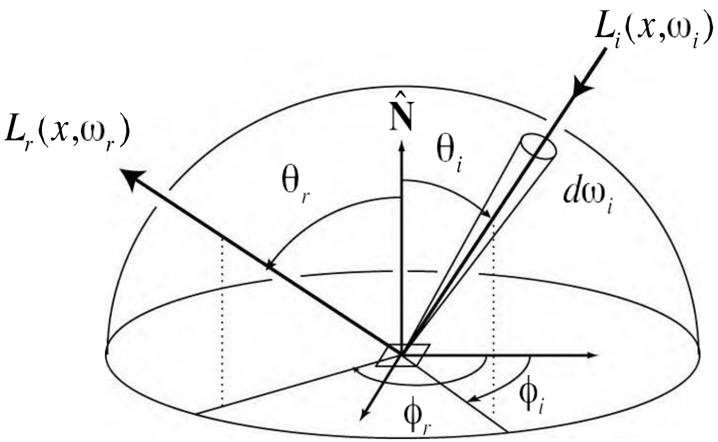


$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r)\, L_i(\mathrm{p},\omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

Challenge: Recursive Equation

Reflected radiance depends on incoming radiance

$$\boxed{L_r(\mathrm{p}, \omega_r)} = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r) \boxed{L_i(\mathrm{p}, \omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



But incoming radiance depends on reflected radiance (at another point in the scene)

The Rendering Equation

Re-write the reflection equation:

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i \, \mathrm{d}\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

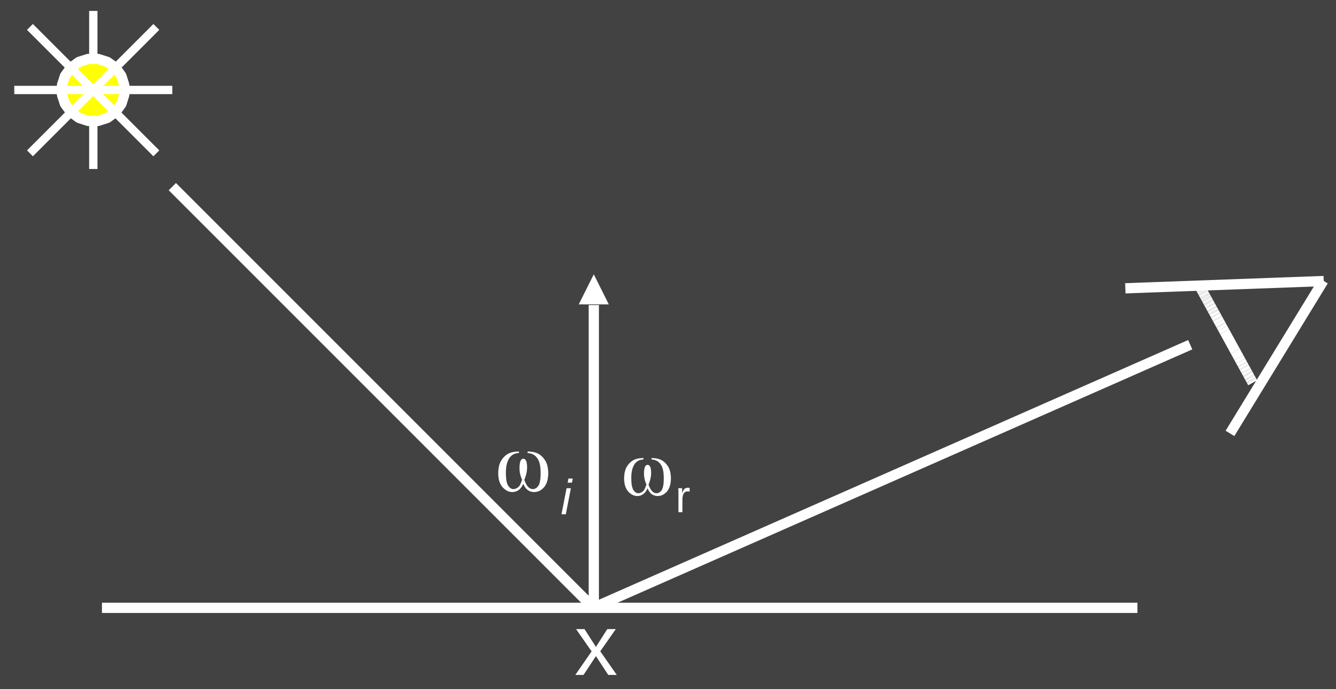
$$\boxed{L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)\, \mathrm{d}\omega_i}$$

How to solve? Next lecture!

Note: now, we assume that all directions are pointing outwards!

Understanding the rendering equation

Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)

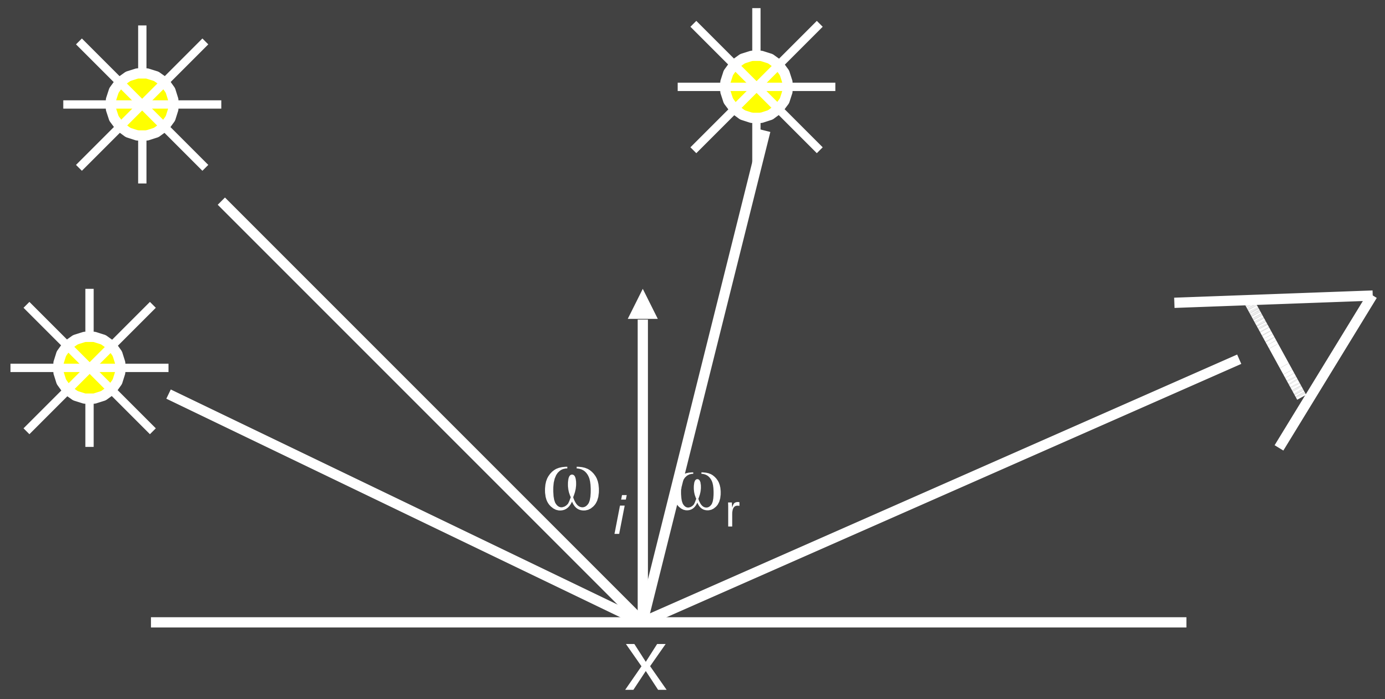
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i, n)$$

Reflected Light (Output Image)

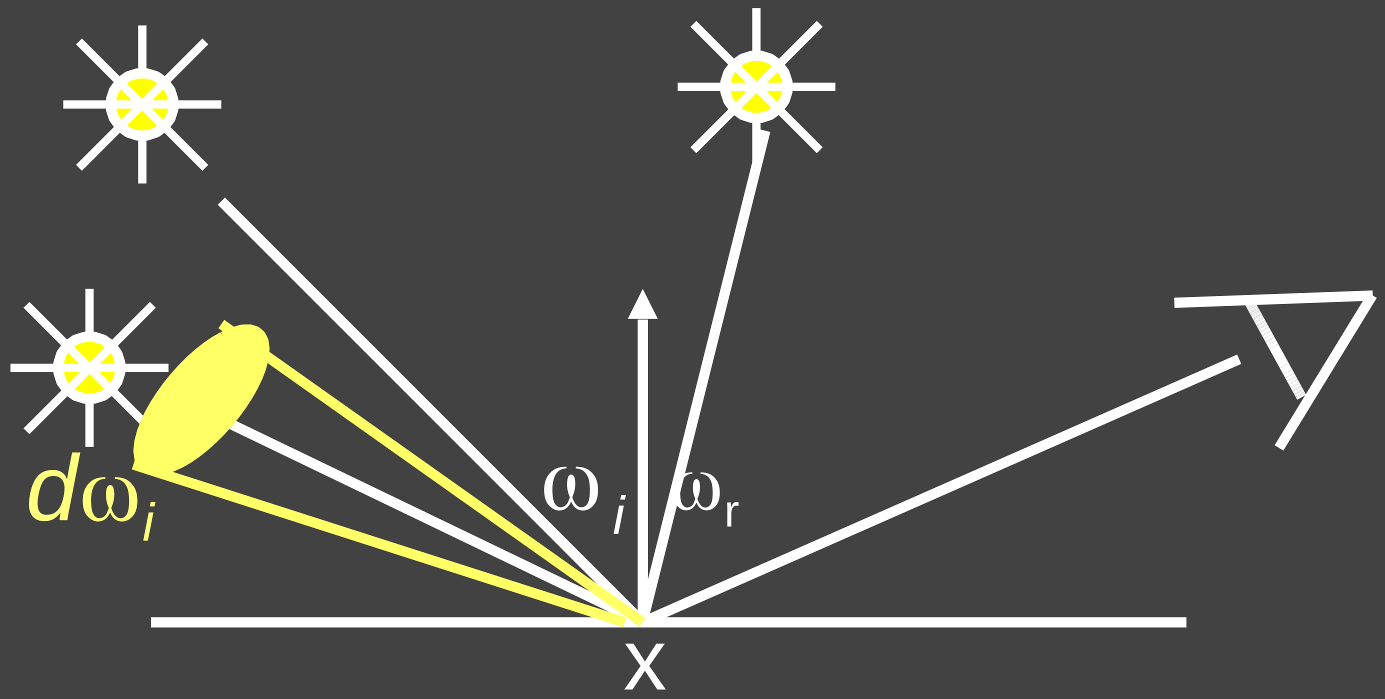
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



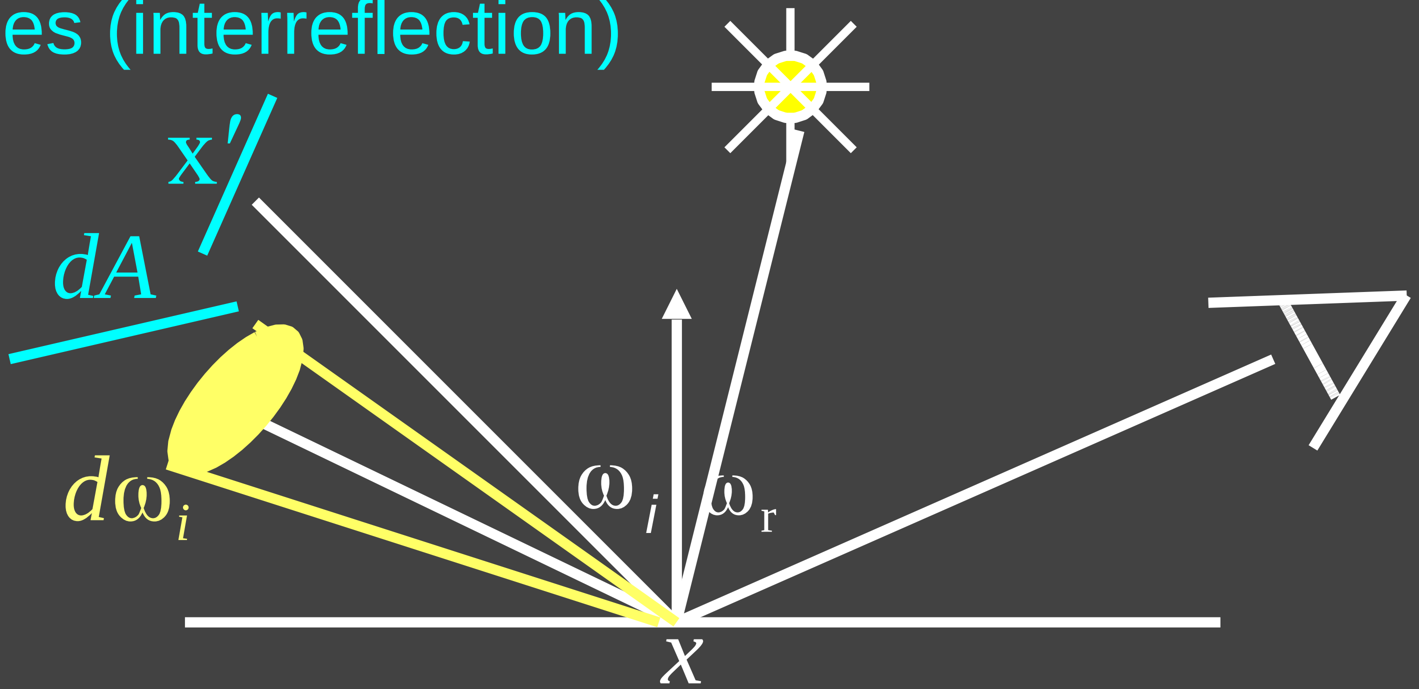
Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ \cos\theta_i\, \mathrm{d}\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\,\boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\,\boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Equation

Kernel of equation
Light Transport Operator

$$l(u) = e(u) + \int l(v) \boxed{K(u,v) dv}$$

$$L = E + KL$$

Can be discretized to a simple matrix equation
[or system of simultaneous linear equations]
(L, E are vectors, K is the light transport
matrix)

Ray Tracing and extensions

- General class numerical Monte Carlo methods

- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2E + K^3E + \ldots$$

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Shading in
Rasterization

Direct illumination


p
One-bounce global illumination (dir+indir)

Two-bounce global illumination


p
Four-bounce global illumination


p
Eight-bounce global illumination


p
Sixteen-bounce global illumination

Probability Review

Random Variables

X random variable. Represents a distribution of potential values

$X \sim p(x)$ probability density function (PDF). Describes relative probability of a random process choosing value x

Example: uniform PDF: all values over a domain are equally likely

e.g. A six-sided die

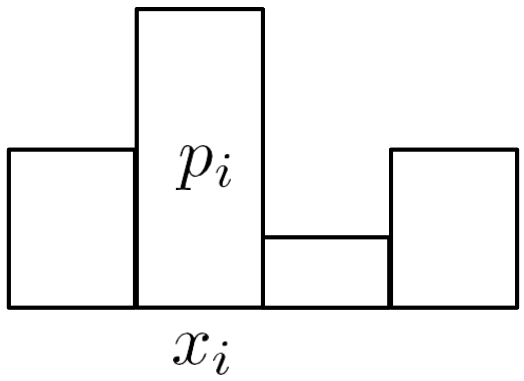
X takes on values 1, 2, 3, 4, 5, 6

$$p(1) = p(2) = p(3) = p(4) = p(5) = p(6)$$

Probabilities

n discrete values $\quad x_i$

With probability $\quad p_i$



Requirements of a probability distribution:

$$p_i \geq 0$$

$$\sum_{i=1}^{n} p_i = 1$$

Six-sided die example: $\quad p_i = \dfrac{1}{6}$

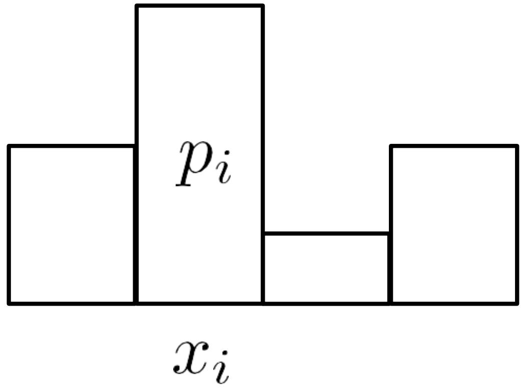
Expected Value of a Random Variable

The average value that one obtains if repeatedly drawing samples from the random distribution.

X drawn from distribution with

n discrete values x_i

with probabilities p_i



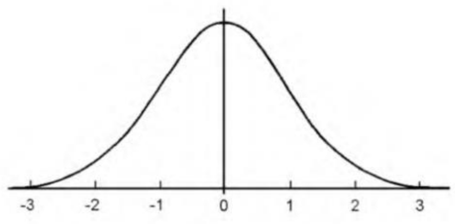
Expected value of X: $$E[X] = \sum_{i=1}^{n} x_i p_i$$

Die example: $$E[X] = \sum_{i=1}^{n} \frac{i}{6}$$

$$= (1+2+3+4+5+6)/6 = 3.5$$

Continuous Case: Probability Distribution Function (PDF)

$$X \sim p(x)$$



A random variable X that can take any of a continuous set of values, where the relative probability of a particular value is given by a continuous probability density function p(x).

Conditions on p(x): $$p(x) \geq 0 \text{ and } \int p(x)\, dx = 1$$

Expected value of X: $$E[X] = \int x\, p(x)\, dx$$

Function of a Random Variable

A function Y of a random variable X is also a random variable:

$$X \sim p(x)$$
$$Y = f(X)$$

Expected value of a function of a random variable:

$$E[Y] = E[f(X)] = \int f(x)\, p(x)\, dx$$

Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)